Tracking 2-D Motion

1. You track a stone launched eastward from a slingshot. Using video analysis, you obtain two position vs time plots: one that tracks its horizontal position and another that tracks its vertical position. You add trend lines to these graphs. They are shown below.
    a. for horizontal position vs time, $y = 32.45x$
    b. for vertical position vs time, $y = -4.9x^2 + 13.11x$

   You promptly interpret them.
    a. for horizontal position vs time, $\mathbf{x_f} = (32.45\ \mathbf{i})t$
    b. for vertical position vs time, $\mathbf{x_f} = (-4.9\ \mathbf{j})t^2 + (13.11\ \mathbf{j})t$

   To make the models a bit easier to write down and manipulate, you change them to…
    a. $\Delta x = 32.45t$
    b. $\Delta y = -4.9t^2 + 13.11t$

   You're now ready to use the two models to work out the details of the stone's trajectory.

   Question: At what time did the stone return to the height from which it began?

   Question: Upon returning to its original height, what is the horizontal displacement ("range") of the stone?

   Question: What is the *horizontal* displacement at 1.10 seconds? What is the *vertical* displacement at 1.10 seconds? What is the displacement (including direction) at 1.10 seconds?

   Do it: Write out both the horizontal and vertical velocity models, in the format $\mathbf{v_f = at + v_i}$.

   Question: What is the *horizontal* velocity at 1.10 seconds? What is the *vertical* velocity at 1.10 seconds? What is the velocity (including direction) at 1.10 seconds?

   Question: At what time does the stone reach its peak? What is the velocity (including direction) of the stone at its peak?

2. You track a soccer ball launched eastward by a player's foot. Video analysis results in the following trend lines:
    a. For horizontal position vs time, $y = 21.45x$
    b. For vertical position vs time, $y = -4.9x^2 + 18x$

Do it: Re-write these two models, using proper notation.

Question: At what time does the soccer ball return to its original height?

Question: What is the range of the ball?

Question: What is the initial velocity (including direction) of the ball?

Question: At what time does the ball reach its peak? At its peak, how high above the ground is the ball?

Do it: Write out both the horizontal and vertical velocity models, in the format $\mathbf{v_f = at + v_i}$.

Question: What is the velocity (including direction) of the soccer ball at 2.5 seconds?

3. A soccer ball is launched from the ground at 20 m/s at an angle of 50° above the horizon.
   a) Find the horizontal and vertical components of the initial velocity.

   b) Write out the horizontal and vertical position models.

   c) Write out the horizontal and vertical velocity models.

   d) At what time does the soccer ball land, i.e. return to its original height?

   e) What is the ball's range?

   f) What is the velocity of the ball at its peak?

   g) What is the velocity of the ball at 2.6 seconds?

4. A baseball is hit by a batter, giving the ball an initial velocity of 42 m/s at 25° above the horizon.
   a) What is the baseball's range at the moment it returns to its original height?

   b) What is the ball's velocity at 3.0 seconds?

5. An out-of-control bus runs off the road and plummets into a ravine. The bus is going 55 mph (24.6 m/s) when it leaves the road, and it falls without resistance some 200 meters to the water below.
   a) Write out both position vs time models.

   b) Write out both velocity vs time models.

   c) Find the horizontal displacement ("range") of the bus.

   d) Find the velocity of the bus at the moment it reaches the water.

   e) Find the velocity of the bus at half its fall time?

6. A smooth metal ball rolls of the edge of a 1.4 meter-high table going 3.4 m/s. What is its range?

   What is the ball's velocity upon reaching the ground?

7. A pencil rolls of the edge of a 0.80 meter-high desk going 0.20 m/s. What is its range?

8. A man throws a grenade toward a building 5 meters away. In the wall of the building is set a window, 2.0 meters above the ground and 1.0 meter tall. The grenade leaves his hand 1.2 meters above the ground, with a velocity of 8.0 m/s at 40° to the horizon. Will the grenade pass through the open window? Justify your answer.

9. A golf ball is launched from the ground at 70 m/s at 42° above the horizon. What is the ball's range?

   At what two times is the golf ball 85 meters above the ground?

   What is the velocity of the ball when it lands?

10. CHALLENGE PROBLEM: A ball is launched from the ground at 38 m/s and lands 78 meters downfield. At what angle was the ball launched?