1. You are standing on train tracks, and a train is approaching you from the west (heading eastward) at 23.3 m/s. Inside the train, a boy is walking toward the back of the train at 0.6 m/s, relative to the train itself. What is the velocity of the boy, relative to you?

The speeds are small, compared to the speed of light. Thus, $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$ can be approximated as simply $u' + v$.

u' = -0.6 **i** (m/s) , **v** = 23.3 **i** (m/s)

u = **u'** + **v** = -0.6 **i** + 23.3 **i** = 22.7 **i** (m/s); the velocity of the boy is 22.7 m/s to the east, relative to you

2. Two cars are at rest, bumper to bumper, facing the same direction. The front car begins driving forward at 15 m/s, while the back car begins driving backward (in reverse) at 10 m/s. Inside the front car, a child throws a pillow toward the back of the car with a speed of 4 m/s, relative to the car. What is the pillow's speed, relative to someone in the back car?

The speeds are relatively small, so we'll use our simplified model. (Remember, we consider less than 50,000,000 m/s to be "small". This is not a rule. It's a personal choice, dependent on the amount of error I'm willing to accept. You may, eventually, decide on a different cutoff. But use my cutoff for now.)

Intuitively, you should know that the answer will be larger than either car's individual speed. The rate at which the cars are separating is greater than either 15 m/s or 10 m/s. Indeed, it is the sum of the two speeds: u' + v = 25 m/s. Because the pillow is thrown (within the front car) toward the back car, the pillow is not moving away from the back car as quickly as the front car itself. Thus, the pillow, previously moving 25 m/s away from the back car, now appears to be moving 25 - 4 = 21 m/s away from the back car. It is unnecessary to state a direction in our answer, as we were not asked for the velocity but the speed.

IMPORTANT: In this case, attempting to fit all of the data into our standard model makes the problem more difficult to solve. Visualizing the situation, and logical thinking (improved by watching others solve problems), leads us to the answer. Don't get so caught up in applying a model that you make an easy problem hard.

3. A motor boat travels down a river, with the current. The water is flowing at 6.2 m/s northward, and the boat is moving at 18.0 m/s relative to the water. How fast is the boat moving, relative to a person walking along the river bank, northward, at 1.0 m/s?

The speed of the boat, relative to the *river bank*, is **u'** + **v** = 18.0 **j** + 6.2 **j** = 24.2 **j** (m/s). The boat would be moving away from the person at 24.2 **j** (m/s) if the person was stationary. As the person is walking at 1.0 **j** (m/s), the boat's apparent speed, relative to the person, is reduced to 23.2 **j** (m/s). It is unnecessary to state the direction, as we are not asked for velocity. Thus, the answer is 23.2 m/s.

4. A woman is skydiving. After deploying her parachute, she falls toward the ground at a steady 8.5 m/s, relative to the ground. She looks up and sees a Coke bottle, which must have fallen from the airplane. It is falling downward, directly toward her, at 20.0 m/s, relative to the ground. (Note that both the woman and the Coke bottle are falling at their terminal velocities. Neither is accelerating.) What is the velocity of the woman, from the point of view of the Coke bottle?

The Coke bottle is moving downward at 20.0 - 8.5 = 11.5 m/s faster than the woman. From the woman's perspective, it is approaching her at - 11.5 **j** (m/s). From the Coke bottle's perspective, the woman is approaching it at 11.5 **j** (m/s).

When adopting a certain perspective, imagine the object tied to that perspective is not moving. Here, we imagine the Coke bottle is not moving. Imagine falling right alongside it. From your point of view, it is not moving. It doesn't move away from you or closer to you. You now have the same perspective as the bottle. And the woman appears to be moving toward you, closing the gap separating you. Since you're above her, she must be moving upward to approach you. That's why we say she is moving at 11.5 m/s upward toward the Coke bottle.

5. A red car and a blue car are at two ends of the flight deck on a large aircraft carrier. Facing each other, the red car drives forward (northward) at 30 m/s while the blue car drives forward (southward) at 25 m/s. Both speeds are relative to the flight deck. Meanwhile, the entire carrier is moving northward at 40 m/s, relative to the water. What is the velocity of the red car, relative to the driver of the blue car?

Does the velocity of the aircraft carrier come into play? No, it doesn't. The cars are approaching each other at the rate of 55 m/s; i.e. the gap between them is shrinking at the rate of 55 m/s. If they began 110 meters apart, they'd crash together after 2 seconds. The driver of the blue car observes the red car to move at 55 m/s northward, or 55 **j** (m/s).

6. A spaceship is heading toward Mars at 80,000,000 m/s, relative to Earth. A missile is shot from the front of the ship, toward Mars, at 12,000 m/s, relative to the ship. What is the velocity of the missile, relative to Earth?

One of our speeds is greater than 50,000,000 m/s, so we must use Einstein's model for calculating relative speeds.

$$u = \frac{12{,}000+80{,}000{,}000}{1+\frac{(12{,}000)(80{,}000{,}000)}{(299{,}792{,}458)^2}}$$ = 80,011,145 m/s away from Earth

If we approximate the speed of light as 300,000,000 m/s, which is easier to remember, we get:

$$u = \frac{12{,}000+80{,}000{,}000}{1+\frac{(12{,}000)(80{,}000{,}000)}{(300{,}000{,}000)^2}}$$ = 80,011,147 m/s away from Earth

This error in this answer is so small (as a %) that most people are willing to accept it. Thus, most people actually use 300,000,000 m/s as the speed of light (in vacuum) in their calculations. Feel free to do this, too.

7. Inside a particle accelerator, a proton is shot forward (say, eastward) at 85% of c, while a second proton is shot toward the first (westward) at 95% of c. What is the speed of one of these protons, relative to the other?

Obviously, we will use Einstein's model, without approximation. I'll also avoid substituting in the value of c, just to make things easier.

$$u = \frac{.85c + .95c}{1 + \frac{(.85c)(.95c)}{c^2}} = \frac{1.8c}{1.8075} = .9958506224c$$

The speed of one proton, from the perspective of the other, is .99585c or 99.585% of c, which is 298,548,506 m/s. If you treated c as 300,000,000 m/s, then .99585c = 298,755,187 m/s. The error is less than 0.1%.

Your answer may differ slightly depending on how many digits of the ".99585…" you kept in your calculator when you multiplied by c. Personally, I kept all of the digits.

8. A spaceship is heading away from Earth at 120,000,000 m/s. From the ship, a missile is launched toward Earth (gasp!) at 20,000 m/s, relative to the ship. What is the velocity of the missile, relative to Earth?

We'll use Einstein's model, without approximation.

$$u = \frac{-20{,}000 + 120{,}000{,}000}{1 + \frac{(-20{,}000)(120{,}000{,}000)}{(299{,}792{,}458)^2}} = \text{119,983,204 m/s away from the Earth}$$

Logical thinking leads us to use a negative sign in the equation. The missile is moving away from the Earth but not as quickly as the spaceship itself. Thus, it has no chance of striking the Earth. (So breathe easy.)

Had we used c = 300,000,000 m/s, we'd have calculated u = 119,983,200 m/s away from the Earth.

9. A spaceship is heading away from the Earth at 78% of c. A lamp attached to the outside of the ship is then turned on, and a beam of light heads toward Earth. What is the speed of the light, relative to Earth? (Prove it mathematically.)

Again, we'll use Einstein's model, without approximation. The speed of the light beam, relative to the spaceship, is of course c or 1c. Its direction toward Earth prompts a negative sign.

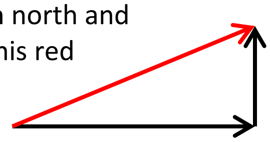
$$u = \frac{-1c + .78c}{1 + \frac{(-1c)(.78c)}{c^2}} = \frac{-.22c}{.22} = -1c$$

The light appears to be approaching the Earth at its one and only speed, c. (The negative sign indicates direction toward the Earth.)

10. A river flows northward at 8.0 m/s. A boat crosses the river, from its west bank to its east bank. The boat travels at a constant 14.0 m/s, relative to the water. (a) What is the velocity of the boat, relative to a tree growing next to the river? (b) If the river is 40 m wide, how long does the boat take to cross? (Assume the boat does not slow down approaching the opposite bank.)

This problem may be a stretch, as we haven't discussed two-dimensional motion. Perhaps some of you worked it out. It was meant to challenge you.

(a) We cannot simply add or subtract our two speeds, as they are in perpendicular directions. Instead, we'll "add" them using the Pythagorean Theorem. This will be discussed in more detail in later sections of the website.



The speed of the boat can be calculated using the Pythagorean Theorem:

$$\sqrt{14^2 + 8.0^2} = 16.1 \text{ m/s}$$

The direction of the boat can be worked out using trigonometry. I'll simply state the answer for now. It's 29.74° north of east.

Our answer for part (a): the velocity of the boat is 16.1 m/s at 29.74°north of east, relative to the tree.

(b) To answer this question, we are going to use the kinematic model $\mathbf{\Delta x} = \mathbf{v}_{avg}\mathbf{t}$.
$\mathbf{\Delta x} = 40\ \mathbf{i}$ (m)
We have three speeds. Which do we choose to work with?
$\mathbf{v} = 14\ \mathbf{i}$ (m/s) , $\mathbf{v} = 8.0\ \mathbf{j}$ (m/s) , or $\mathbf{v} = 14\ \mathbf{i} + 8.0\ \mathbf{j}$ (m/s)
The last of the three velocities listed above is equivalent to 16.1 m/s at 29.74° north of east.
Which of these three velocities do we substitute into our model $\mathbf{\Delta x} = \mathbf{v}_{avg}\mathbf{t}$?
Because $\mathbf{\Delta x}$ is in the direction of $\mathbf{i}$, so must be our velocity. So we choose $\mathbf{v} = 14\ \mathbf{i}$ (m/s).

Thus, $\mathbf{\Delta x} = \mathbf{v}_{avg}\mathbf{t}$ leads to $t = \mathbf{\Delta x} / \mathbf{v}_{avg} = 40\ \mathbf{i} / 14\ \mathbf{i} =$ 2.86 seconds. Conceptually, this indicates that the rate at which the river is flowing has no effect on how long it takes to cross. It simply affects how far downstream the boat will land. Can you work out how far downstream it lands?