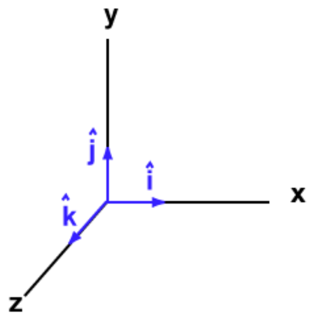
$\Delta x = v_{avg}t$ ⇒ $\Delta x = v_it + \frac{1}{2}at^2$

$v_f = at + v_i$ ⇒ $v_f^2 = v_i^2 + 2a\Delta x$

The standard 4 kinematic equations.



Use the 4 equations (models) above to answer the following questions.
(Note: Remember to use negative signs, where appropriate.)

For most problems, there is more than one way to solve for the desired variable. It's okay if you chose a different model than I did, as long as you got the same result.

1. A bicyclist is going 6.2 m/s eastward. He accelerates eastward at a constant 0.8 m/s/s for 5.0 seconds. Find the cyclist's (a) final velocity and (b) displacement.

$\mathbf{v_i}$ = 6.2 **i** (m/s) , **a** = 0.8 **i** (m/s/s) , t = 5.0 (s)

(a) $\mathbf{v_f}$ = ? , let's use $\mathbf{v_f = at + v_i}$
$\mathbf{v_f}$ = (0.8 **i**)(5.0) + (6.2 **i**) = 10.2 **i** (m/s)
(b) $\mathbf{\Delta x}$ = ? , let's use $\mathbf{\Delta x = v_it + \frac{1}{2}at^2}$
$\mathbf{\Delta x}$ = (6.2 **i**)(5.0) + ½(0.8 **i**)(5.0^2) = 41.0 **i** (m)

2. A car is driving northward at 14.1 m/s and accelerates at a constant rate to 25.0 m/s northward. If the car's displacement during this acceleration is 70.4 meters northward, (a) how long was the car accelerating and (b) what was its rate of acceleration?

$\mathbf{v_i}$ = 14.1 **j** (m/s) , $\mathbf{v_f}$ = 25.0 **j** (m/s) , $\mathbf{\Delta x}$ = 70.4 **j** (m)

(a) t = ? , let's use $\mathbf{\Delta x = v_{avg}t = \frac{v_i+v_f}{2}t}$
t = 2$\mathbf{\Delta x}$ / ($\mathbf{v_i + v_f}$) = 2(70.4 **j**) / (14.1 **j** + 25.0 **j**) = 3.60 (s)
(b) **a** = ? , let's use $\mathbf{v_f^2 = v_i^2 + 2a\Delta x}$
a = ($\mathbf{v_f^2 - v_i^2}$) / 2$\mathbf{\Delta x}$ = ((25.0 **j**)^2 - (14.1 **j**)^2) / 2(70.4 **j**) = 3.03 **j** (m/s/s)

3. A car, starting from rest, accelerates westward at 1.35 m/s/s for 3.0 seconds. What is its displacement during this time?

$\mathbf{v_i}$ = 0 (m/s) , **a** = -1.35 **i** (m/s/s) , t = 3.0 (s)

$\mathbf{\Delta x}$ = ? , let's use $\mathbf{\Delta x = v_it + \frac{1}{2}at^2}$
$\mathbf{\Delta x}$ = (0)(3.0) + ½(-1.35 **i**)(3.0^2) = -6.075 **i** (m)

4. An airplane is heading northward at 260 m/s. To slow down, it accelerates southward at 40.0 m/s/s. (a) How much is its velocity reduced over a displacement of 200 meters northward? (b) How long does it take to slow to 180 m/s northward?

$\mathbf{v_i}$ = 260 **j** (m/s) , **a** = -40.0 **j** (m/s/s)

(a) $\mathbf{\Delta x}$ = 200 **j** (m) , $\mathbf{v_f}$ = ? , let's use $\mathbf{v_f^2 = v_i^2 + 2a\Delta x}$

$\mathbf{v_f} = \sqrt{(200\,\mathbf{j})^2 + 2(-40.0\,\mathbf{j})(200\,\mathbf{j})}$ = 227.2 **j** (m/s), so the velocity is reduced 32.8 **j** (m/s)

(b) $\mathbf{v_f}$ = 180 **j** (m/s) , t = ? , let's use $\mathbf{v_f = at + v_i}$

t = ($\mathbf{v_f}$ - $\mathbf{v_i}$) / **a** = (180 **j** - 260 **j**) / -40.0 **j** = 2.0 (s)

5. A car begins from rest and accelerates southward at a constant rate for 4.8 seconds. Over this period of time, its average velocity is 12 m/s southward. What is the car's rate of acceleration?

$\mathbf{v_i}$ = 0 (m/s) , t = 4.8 (s) , $\mathbf{v_{avg}}$ = 12 **j** (m/s)

a = ? , let's use $\mathbf{\Delta x = v_{avg}t}$ and $\mathbf{\Delta x = v_i t + ½at^2}$

$\mathbf{\Delta x}$ = (12 **j**)4.8 = 57.6 **j** (m)

also, $\mathbf{\Delta x = v_i t + ½at^2}$ which leads to 57.6 **j** = ½**a**(4.8^2) and **a** = 5.0 **j** (m/s/s)

6. A snowmobile is heading toward a tree at some particular speed. The operator releases the throttle and the machine begins to slow at the rate of 4.0 m/s/s. If the snowmobile comes to rest in 35 m, several meters in front of the tree, what was its initial speed?

a = -4.0 **i** (m/s/s), the direction is assumed , $\mathbf{\Delta x}$ = 35 **i** (m) , $\mathbf{v_f}$ = 0 (m/s)

$\mathbf{v_i}$ = ?, let's use $\mathbf{v_f^2 = v_i^2 + 2a\Delta x}$

$\mathbf{v_i} = \sqrt{\mathbf{v_f^2 - 2a\Delta x}} = \sqrt{(0)^2 - 2(-4.0\,\mathbf{i})(35\,\mathbf{i})}$ = 16.7 **i** (m/s); the speed is just 16.7 m/s (no direction)

7. Two cars are 400.0 meters apart and are facing one another. Imagine they're on a single-lane road. Beginning simultaneously, the red one travels forward at a constant speed of 18 m/s, and the blue one travels forward at a constant speed of 26 m/s. After 3.5 seconds, what is the distance between the cars?

We'll model the cars separately, and then use both models to answer the question.

For the red car, $\mathbf{v_{avg}}$ = 18 **i** (m/s), where the direction is assumed, t = 3.5 (s)

$\mathbf{\Delta x}$ = ? , let's use $\mathbf{\Delta x = v_{avg}t}$, $\mathbf{\Delta x}$ = (18 **i**)(3.5) = 63 **i** (m)

For the blue car, $\mathbf{v_{avg}}$ = -26 **i** (m/s), where the direction is opposite the red car, t = 3.5 (s)

$\mathbf{\Delta x}$ = ? , let's use $\mathbf{\Delta x = v_{avg}t}$, $\mathbf{\Delta x}$ = (-26 **i**)(3.5) = -91 **i** (m)

The cars cover a combined (63 + 91) 154 m during the period. In other words, they are now 154 m closer to one another, which means they are 400.0 - 154 = 246 m apart.

8. An airplane increases its velocity from 20 m/s to 35 m/s westward while undergoing a displacement of 515 meters westward. What is the airplane's acceleration during this period?

$\mathbf{v_i}$ = -20 **i** (m/s) , $\mathbf{v_f}$ = -35 **i** (m/s) , **Δx** = -515 **i** (m)

a = ? , let's use $\mathbf{v_f^2 = v_i^2 + 2a\Delta x}$

$\mathbf{a = (v_f^2 - v_i^2) / 2\Delta x} = \frac{(-35\,\mathbf{i})^2 - (-20\,\mathbf{i})^2}{2(-515\,\mathbf{i})} = -0.801\,\mathbf{i}$ (m/s/s)

9. A bus is traveling eastward at 8.20 m/s when it begins to accelerate at 0.55 m/s/s eastward. How long does it take for the bus to travel 61.4 meters eastward?

$\mathbf{v_i}$ = 8.20 **i** (m/s) , **a** = 0.55 **i** (m/s/s) , **Δx** = 61.4 **i** (m)

t = ? , let's use $\mathbf{\Delta x = v_i t + ½at^2}$

61.4 **i** = (8.20 **i**)t + ½(0.55 **i**)t^2 , which must be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Don't confuse this "a" with acceleration.

To do this, we rearrange the equation to match the format $\mathbf{ax^2 + bx + c = 0}$.

I'm going to drop the **i**'s to make it less messy (but I wouldn't drop negative signs if we had them).

$0.275t^2 + 8.20t - 61.4 = 0$ … which means 0.275 is our a term, 8.20 our b term, and -61.4 is c

$t = \frac{-8.20 \pm \sqrt{8.20^2 - 4(0.275)(-61.4)}}{2(0.275)} = \frac{-8.20 \pm 11.61}{0.55}$ = 6.2 and -36.0 (s)

Now, in a math course you would state both solutions of this equation. Not so in physics. Only one of these solutions has any physical significance. A negative time is not meaningful, so we ignore it. The negative time does not belong in our model. So the answer to the original question is 6.2 seconds.

10. A bus is traveling eastward at 8.20 m/s when it begins to accelerate at 0.55 m/s/s westward. What is its velocity upon covering an additional 30 m eastward?

$\mathbf{v_i}$ = 8.20 **i** (m/s) , **a** = -0.55 **i** (m/s/s) , **Δx** = 30 **i** (m)

$\mathbf{v_f}$ = ? , let's use $\mathbf{v_f^2 = v_i^2 + 2a\Delta x}$

$\mathbf{v_f} = \sqrt{(8.20\,\mathbf{i})^2 + 2(-0.55\,\mathbf{i})(30\,\mathbf{i})} = 5.85\,\mathbf{i}$ (m/s)