Idealization and Abstraction: A Framework

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Preprint

When, in the theoretical practice of science, we put forward theories, construct models, and write down laws, whether we are doing microeconomics, population genetics, or cosmology, we often seem to be describing, picturing, or making claims about systems which bear only a distant relation to the systems we actually encounter in the world around us. What is more, we often take ourselves to be doing just that. Consider, for example, the following methodological declaration by Noam Chomsky, from a passage which appears at the very beginning of his seminal *Aspects of the Theory of Syntax*:

> Linguistic theory is concerned primarily with an ideal speaker-listener, in a completely homogeneous speech-community, who knows its language perfectly and is unaffected by such grammatically irrelevant conditions as memory limitations, distractions, shifts of attention and interest, and errors (random or characteristic) in applying his knowledge of the language in actual performance. This seems to me to have been the position of the founders of modern general linguistics, and no cogent reason for modifying it has been offered. (Chomsky, 1965, pp. 3-4)

Chomsky is quite explicit about the chances of encountering such an ideal speaker-listener:

> We...make a fundamental distinction between *competence* (the speaker-hearer's knowledge of his language) and *performance* (the actual use of language in concrete situations). Only under the idealization set forth...is performance a direct reflection of competence. In actual fact, it obviously could not directly reflect competence. A record of natural speech will show

> numerous false starts, deviations from the rules, changes of plan in mid-course, and so on. (*ibid*., p. 4)

Similar remarks can be found in Section 1.4 of Robert A. Granger's engineering text *Fluid Mechanics*, a section entitled "Fundamental Idealizations," and they are so apposite, and so explicit, that I will quote them in full:

> Theoretical fluid mechanics is an attempt to predict the behavior of real fluid motions by solving boundary value problems of either appropriate partial differential equations or integral equations.... In deriving the well-set boundary value equations, we postulate certain boundary and "inner" conditions which inevitably dictate the final form of the solution. With such a set of equations, we can solve few problems. Analytic solutions are impossible, numerical solutions are inappropriate, and nothing appears to work. Only the simplest fluid flow problems can be solved.
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> Therefore, we introduce idealizations into the problems. We might assume that the fluid is independent of time, reasoning that the disturbances are of secondary importance. We could assume that the fluid is ideal [i.e., has zero viscosity], when in fact no known fluid is ideal. But because the viscosity may be small, much smaller than, say, for water, the idealization will yield solutions that are acceptable. What else might we assume? The possibilities are endless. For example, we could assume the flow is (a) symmetric, (b) incompressible, (c) not rotating, (d) one-dimensional, (e) continuous, (f) isothermal, (g) isobaric, (h) adiabatic, (i) reversible, (j) homogeneous, etc. The flow of course, may be none of these, for all are idealizations. (Granger, 1995, p. 17)

Both Chomsky and Granger are drawing attention to specific ways in which the real systems in their respective domains of inquiry are knowingly and systematically misrepresented: no real speaker-listener is unaffected by memory limitations, and no real fluid has zero viscosity or is, strictly speaking, incompressible. Representations also omit features of the systems under study without thereby misrepresenting them, of course: any real speaker-listener has a specific height and weight, and real fluids are particular colors. Putting these two points somewhat colorfully, we might say that when, in the various sciences, we theorize about a certain class of systems, we habitually lie about some aspects of the systems in question, and entirely neglect to mention others.

I intend to take this distinction between misrepresentation and mere omission as fundamental, and to suggest that we organise our terminology around it. On the regimentation of usage I am thus proposing, the term 'idealization' applies, first and foremost, to specific respects in which a given representation misrepresents, whereas the term 'abstraction' applies to mere omissions.[1] [2] One of my two primary aims in this paper is to develop this way of distinguishing between idealization and abstraction further; the second is to provide some characterization of the precise forms each can take in various sorts of scientific representation. These aims go hand in hand, and I will pursue them together. The overall intention is to provide an account of certain areas of the conceptual terrain, and some recommendations about how best to navigate it.[3]

My starting point, then, is the suggestion that we should take idealization to require the assertion of a falsehood, and take abstraction to involve the omission of a truth.[4] In the next section I will be qualifying, extending, and elaborating upon this

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[1] The examples of omissions just given may make abstractions in this sense seem relatively uninteresting; the discussion of abstraction and relevance, in section 2 below, will help to dispel that impression.

[2] Nancy Cartwright carves things up somewhat similarly in *Nature's Capacities and their Measurement.* One important difference, however, is that Cartwright seems to build into the notion of abstraction she employs at least two features that I do not wish to build into mine: (i) that it is causal factors which we are focussing on when we "subtract" various other features of the situation, and (ii) that the "material in which the cause is embedded" is subtracted when we formulate an abstract law. Cartwright also claims that in the case of laws which are abstract in her sense, "it makes no sense to talk about the departure of the…law from truth;" this is not something that will necessarily be true of laws which are abstract in the sense I hope to characterize below. Note also that in the passages in which Cartwright characterizes her notions of idealization and abstraction, respectively, she speaks for the most part (although not exclusively) of idealized models and abstract laws. I should thus emphasize that on the proposal I wish to offer, idealizations and abstractions each appear plentifully both in models and in laws. (Cartwright, 1989, pp. 187-8.)

[3] There is by now a considerable body of work on idealization and abstraction in the philosophical literature; indeed, a significant part of that literature is represented in the series containing this volume (including much important work which has been done in continental Europe). It is no part of my ambition in this paper to provide a comprehensive discussion of the range of approaches which have been developed by various authors. Rather, my aim is to develop one specific proposal and show that some useful work can be done with it. (For an introduction to some of the European literature, see, for example, Leszek Nowak's "The Idealizational Approach to Science: A Survey" (1992).)

[4] The distinction I have in mind thus loosely parallels the mediaeval legal distinction between *suggestio falsi* and *suppressio veri*, except that the phrase "suggestion of a falsehood" is rather euphemistic in the case of many idealizations, and that furthermore, at least on a good day, no one is deceived by scientific idealizations and abstractions. (I am indebted to Alan Code for some useful information concerning the

proposal for regimenting the terminology. Several immediate points of clarification are called for, however. First, not all misrepresentations can properly be called idealizations; nor, perhaps, are all omissions abstractions. What more is involved in either case will be considered later (in section 2). Secondly, the sort of omission I have in mind (a "mere omission") is such that the category of idealizations and the category of abstractions are mutually exclusive (although happily not exhaustive). If a model of a particular fluid flow represents the flow as irrotational when it is not, we can in one sense correctly say that the model omits the rotation involved in the flow. However, such a model omits a certain feature of the real system in a way which involves misrepresenting how things stand in that respect; on the proposal I am putting forward, however, abstractions involve omission *without* misrepresentation. Omission in this restricted sense is, so to speak, a matter of complete silence. It follows straightforwardly that, with respect to a particular feature of a certain real system, a given representation can contain an idealization, or an abstraction, or neither, but it cannot contain both.[5]

Thirdly, it is important to be clear on the intended force of the proposal at hand. I am not hoping to capture the essence of scientific or philosophical usage *in toto*, for the simple reason that there is no single common usage of the terms in question. My proposal conflicts straightforwardly with the usage of some authors; indeed, both of the phenomena I have in mind have been referred to by each of the labels in question.[6]

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mediaeval terms.) On that note, it is worth emphasizing that idealization need not involve the assertion of a falsehood *on our part*—it is enough for a model to contain an idealization that *it* misrepresents the world in some respect. We can use idealized models without believing the lies they tell.

[5] That is not to say, of course, that a given representation cannot idealize some aspects of a system and abstract away from others.

[6] See, for example, Ernan McMullin's "Galilean Idealization" (1985), an important contribution to the discussion of this topic. Whilst recognizing that "[t]he term, 'idealization,' itself is a rather loose one," McMullin opts for taking it to "signify a deliberate simplifying of something complicated…with a view to achieving at least a partial understanding of that thing." He then adds: "[Idealization] may involve a distortion of the original or it can simply mean a leaving aside of some components in a complex in order to focus the better on the remaining ones" (p. 248). In my terms, then, McMullin uses the label 'idealization' for both idealization and abstraction. Interestingly, however, on the next page, in characterizing what he calls "mathematical idealization," of which omission is the primary characteristic, McMullin shows a momentary preference for the *other* term: "Aristotle…, of course, separate[d] mathematics quite sharply from physics, partly on the basis of the degree of abstraction (or idealization) characteristic of each….[M]athematics abstracts…from qualitative accidents and change. *A physics that borrows its principles from mathematics is thus inevitably incomplete* as physics, *because it has left aside the qualitative richness of Nature. But it is* not *on that*

Nonetheless, this proposed way of regimenting the terms does capture a philosophically important distinction between two different sorts of features scientific representations typically have (a distinction that has been recognized and employed by many authors), and it does mesh with the usage of some philosophers and, I would claim, with much ordinary scientific usage.

To make a fourth and final point of clarification regarding the basic proposal, let me group misrepresentations and mere omissions together under the heading 'RI's' (for 'representational imperfections,' which is uncomfortably loaded in its connotations—but then this is merely a temporary device). What I wish to emphasize is that in proposing that we regard one particular semantic distinction amongst RI's as fundamental (the distinction between misrepresentations and mere omissions), I do not mean to deny the importance of various other ways in which we might distinguish amongst them. There are certainly significant epistemological lines to be drawn: between RI's which we know to be RI's and those we do not; between RI's where we know the truth of the matter and those where we do not; and between RI' whose effects we can predict and those whose effects we cannot, for example. We might also distinguish amongst RI's with respect to the source of our knowledge concerning them—at the most coarse-grained level, for example, with respect to whether the source is theory or experiment. Alternatively, we might find it useful to draw distinctions along lines which reflect the causal relevance of the features which we have either misrepresented or distorted. Each of these distinctions will be important and useful in some philosophical contexts, as will yet others, and some of them have been given positions of central importance in other approaches to these issues. Indeed, we will return to some of the distinctions I have just mentioned at various points later in this essay. My claim, however, is that tying the terms

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account distortive, as far as it goes" (p. 249, my emphasis). McMullin's "mathematical idealization" would by my lights clearly be classed as a form of abstraction, rather than idealization. (One might, on the other hand, read McMullin's distinction between "formal" and "material" idealization (pp. 258-9) as similar to my distinction between idealization and abstraction, in spirit at least. See n. 35.)

'idealization' and 'abstraction' to the semantic distinction between misrepresentation and omission provides a good starting point if one wishes to construct a larger framework which illuminates the various ways we think about imperfection in scientific representation, and which enables us to articulate certain ideas in greater detail. Just such a framework will be developed in the remainder of the paper.

We need to begin by thinking about the sorts of things which contain abstractions and idealizations. Models, laws, and theories perhaps come first to mind, but we might add explanations, predictions, calculations, graphs, and diagrams to the list. (No doubt we could go on.) In what follows, I will focus largely on the first two sorts of item, models and laws. The hope is that if we can say what it means for models and laws, respectively, to involve idealizations, and what it means for them to involve abstractions, then much of the rest will follow. Scientific explanations and predictions will, at least in many cases, involve idealizations or abstractions just because they employ laws, models, or theories which do, and the same can be said for calculations performed in the service of other ends.[7] I will take it that graphs and diagrams are implicitly covered in my discussion of models, for they are either models themselves, or, perhaps, means of presenting models.

That leaves only theories. The relation between laws, models, and theories has been a much-debated issue in the philosophy of science for at least the last thirty years or so. The older, "syntactic" view typically regarded theories as deductively closed sets of sentences in a formal language, or at least as rationally reconstructable along such lines; the language itself was often regarded as only partially interpreted. Some especially important sentences, or (on some views) all the sentences which make up the

[7] Consider, for example, calculations performed to check the internal consistency of a theory, or to check the equivalence of what are intended to be two formulations of the same theory. (Of course, I do not wish to suggest that all explanations or all predictions involve calculation.)

theory, are then taken to state its laws.[8] The newer "semantic" view, on the other hand, is usually characterized as holding that theories are collections of models.[9] The only stand I wish to take on these issues is, in the current climate, quite a minimal one, and it is that theories tend to involve both laws and models as important components.[10] If that is right, then in characterizing the ways in which both abstractions and idealizations can occur in laws and models, we can hope to gain a considerable purchase on the ways in which they occur in scientific theories.

The structure of the rest of this paper is thus as follows: I discuss idealization and abstraction in models in sections 1–4. I begin by focussing on models of particular systems, and offer a more precise account of the basic distinction I have drawn as it applies in that setting (section 1). I then consider what else we might have in mind when we speak of idealization and of abstraction, in addition to misrepresentation and mere omission respectively (section 2). I go on to extend the account to models of kinds of systems (section 3), and to talk of degrees of idealization and abstraction in models, and talk of idealization and abstraction as processes (section 4). Then, in sections 5-9, I turn to laws. After a few necessary preliminaries (section 5), I distinguish three different ways in which idealization can occur in laws and our employment of them (sections 6-8), and close by saying a few brief words about abstraction in laws (section 9).

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[8] This view is still quite often called the "Received View," even though the label is, by now, highly anachronistic. For a statement of the syntactic view, see Carnap (1970); for well-known critiques, see Suppe (1972) and (1974 a), and Putnam (1979).

[9] This slogan can be rather misleading, however. See Jones (forthcoming a) for further discussion. The semantic view is presented and developed in different ways in: Suppes (1957, ch. 12), (1960), (1967), (1974); van Fraassen (1970), (1972), (1980, ch. 3), (1987), (1989, ch. 9); Suppe (1967), (1974 a), (1989); and Giere (1988).

[10] Although proponents of the semantic view typically wish to draw our attention towards models, and away from such relatively linguistic items as laws (or law statements), they are certainly not aiming to eliminate the latter notion. Frederick Suppe, one of the earliest and most well-known proponents of the semantic view, devotes a considerable part of his extended treatise on theory structure, *The Semantic Conception of Theories and Scientific Realism*, to providing an account of various types of law; indeed, the work contains more explicit discussion of the nature of laws than of the nature of models (Suppe, 1989). (Suppe's aim is to give an account of laws which avoids tying them too closely to sentences in any particular language.) Even van Fraassen's extended attack on laws in his *Laws and Symmetry* (1989) is primarily directed at a number of philosophical theses about laws and the role they play in science and epistemology; he does not deny that Ohm's law, Boyle's law, the Hardy-Weinberg law, or Schrödinger's equation play *some* sort of important role in the theoretical practice of their various sciences, and in the theories with which they are associated.

1. Models: The basic distinction

Before we attempt to say more about what it means to talk about idealization and abstraction in models, it will be useful, particularly in the current philosophical climate, to say something about models themselves. The term 'model' is used in a wide variety of ways in the philosophy of science, and in science itself. Distinguishing the notions which go by that name and relating them to one another, although crucial for some philosophical purposes, is a lengthy and complex matter. Fortunately, it is not something we need to accomplish in any detail here; a few broad outlines will suffice.[11]

On some uses of the term, a model is a model of a set of sentences, in the sense that it makes the sentences in the set true,[12] often by providing them with an interpretation on which they turn out true. On other uses, a model is a model of an object, system, event, or process, in the sense that it represents, or is used to represent that object, system, event, or process as having certain features, behaving in certain ways, and so on.[13] (For the sake of brevity, I will hereafter speak simply of systems and features.) It is only models in the latter sense which will concern us here; for our purposes, a model is, first and foremost, a representation.[14] One can go on to distinguish at least three notions of model as representation in the philosophy of science and in the sciences themselves, the differences lying in the kind of object which does the representing in question: a mathematical structure, such as a vector space with a trajectory running through it (i.e., a function mapping points in some interval on the real

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[11] For a taxonomy of some of the central notions of model abroad in the philosophy of science, a discussion of the suitability of the various notions to certain tasks, a critique of the semantic view (at least in some of its incarnations), and a case for taking a somewhat different view of theory structure, as well as references to the relevant literature, see Jones (forthcoming a); see also Jones (forthcoming b).

[12] Or true-in-the-model, in certain logical contexts. See Jones (forthcoming a); and thanks to Charles Chihara for drawing my attention to this point.

[13] See Frisch (1998) and Jones (forthcoming a) for further discussion of the distinction between models as truth-makers and models as representations.

[14] In principle, of course, one and the same object might serve as a model in both senses. If and when this does occur, then we will be focussing on the object's role as representation, rather than its role as truth-maker. (Something like this situation arises in the version of the semantic view van Fraassen presented in "On the Extension of Beth's Semantics of Physical Theories" (1970), in which the state space for a given system plays a role in the formal semantics for the language of the theory. As I understand that approach, however, it would be an oversimplification to say that one and the same object functions as both representation and truth-maker.)

line, representing times, to elements of the vector space, representing states of the modeled system); a set of propositions; or a physical object, such as an engineer's scale model of a bridge, or an electrical circuit used to represent the behavior of an acoustical system.[15] These differences, however, are at least initially unimportant from our present point of view. We can make a start on the job of clarifying the notions of abstraction and idealization simply by thinking of a model as something which represents a given system as having various features.

In fact, this is insufficiently general, for in addition to models of particular systems, such as models of the 1989 Loma Prieta earthquake or the Big Bang, there are also models of kinds of systems, such as Bohr's model of the hydrogen atom or a classical model of electromagnetic radiation *in vacuo*.[16] The strategy I will adopt, however, is as follows: I will take as basic the notion of a specific idealization contained in a model of a particular system (i.e., an aspect of the model which idealizes the system in some specific respect), and the parallel notion of a model of a specific abstraction present in a model of a particular system. After spending some time providing an account of these two notions (in this section and the next), it will then be a relatively quick matter to extend the account into certain neighboring areas: talk about a model of a kind idealizing and abstracting in specific respects (in section 3); the classification of a model as idealized, highly idealized, or an idealization, or as abstract, highly abstract, or an abstraction; comparative judgements about the extent to which various models idealize a given system, or kind of system, both in a given respect and overall, and about the degree of abstractness of various models; and talk about idealization and abstraction as component processes in scientific theorizing (in section 4).[17]

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[15] For more on these three notions of model as representation, see Jones (forthcoming a).

[16] We also sometimes speak of using one and the same model to represent different particular systems, or even different kinds of system, on different occasions. It worth noting that it is easier to make sense of such talk when the model in question is an abstract mathematical structure or a concrete physical object that when it is a set of propositions.

[17] To say that this will be a quick matter should not to be taken to suggest that there will be no open questions by the time we are done.

Let us begin, then, by considering a simple example of the use of a model to represent a particular system.

Suppose that on some particular afternoon a certain cannon has been wheeled onto an open plain and fired. In the attempt to predict where the cannonball will land, or perhaps to explain why it lands where it does, we might construct a model of the system along the following lines.[18] We assume that the path of the cannonball is contained entirely within a plane perpendicular to the ground, and we choose a pair of Cartesian axes to coordinatize this plane in such a way that the x-axis lies along the ground, and the y-axis points vertically upwards. The axes are also chosen so that the cannon is situated at the origin, and so that the cannonball, when fired, moves in such a way that its x-coordinate increases. The cannonball has an initial velocity of v when fired, and is, at that initial moment, moving at an angle $\alpha$to the x-axis:

[Figure 1]

[18] The example dates back to Niccolò Tartaglia's *Nova Scientia* of 1537, the first two books of which are translated in Drake and Drabkin (1969), but the modelling of the situation presented here is, of course, far more modern, and typical of treatments to be found in contemporary introductory-level textbooks in classical mechanics. Tartaglia, incidentally, rather charmingly distanced himself from the choice of example in the later *Quesiti* (1546): "I…have never made any profession of or delighted in shooting of any kind—artillery, arquebus, mortar, or pistol—and never intend to shoot" (Drake and Drabkin (1969), p. 98). The opening of the *Nova Scientia*, in which Tartaglia describes the history of his work in a letter of dedication to the Duke of Urbino, contains an expression of more emphatic, if somewhat partisan feelings on the matter: "[O]ne day I fell to thinking it a blameworthy thing, to be condemned—cruel and deserving of no small punishment by God—to study and improve such a damnable exercise, destroyer of the human species, and especially of Christians in their continual wars. For which reasons…not only did I wholly put off the study of such matters and turn to other studies, but I also destroyed and burned all my calculations and writings that bore on this subject" (*ibid*., p. 68).

After it has been fired from the cannon, we suppose that the cannonball moves under the sole influence of gravity, which exerts a force vertically downwards with a magnitude of mg (the mass of the cannonball, m, multiplied by a certain constant, $g = 9.8$ m/s^2) throughout the motion. Thus we have:

$$F_y = -mg \tag{1}$$

for the force in the y direction, and

$$F_x = 0 \tag{2}$$

for the force in the x direction. Newton's second law of motion gives us

$$F_y = m\frac{d^2y}{dt^2} \tag{3}$$

and

$$F_x = m\frac{d^2x}{dt^2} \tag{4}$$

so we get

$$\frac{d^2y}{dt^2} = -g \tag{5}$$

and

$$\frac{d^2x}{dt^2} = 0 \tag{6}$$

Given specific values of v and α, it is then a matter of simple integration to calculate the time of flight of the cannonball, the distance from the cannon at which it will land, the maximum height it will reach, and other features of its trajectory. And it is not much more difficult to show that, according to this model, the range of the cannon is maximized for a given v when $\alpha = 45°$.[19]

The sort of model I have just presented contains a number of idealizations, in the sense I am attempting to characterize.[20] According to the model, for example, the gravitational force due to the Earth has the same magnitude and direction at all points on the cannonball's trajectory, whereas in fact both the magnitude and the direction of that force will vary. According to the model, only the Earth exerts a gravitational force on the cannonball, whereas in fact the moon, the sun, and every other massive body in the universe interacts gravitationally with it. What is more, the model says that the only force acting on the projectile is gravitational, whereas in reality the cannonball is subject to air resistance, the influence of breezes, and even forces due to the photons which impinge upon it. And, as a final example, note that as the x-axis lies along the ground (which is putatively flat), the cannonball does not really begin the untrammeled part of its journey at the origin (i.e., at the point (0, 0)), simply because the mouth of the cannon is at a certain height above the ground.

Here we have, then, a number of discrepancies between the way the model (or sort of model) in question represents the modeled system as being, and the way the system really is. It is perfectly in keeping with at least some standard usage to call each of those ways in which the model misrepresents the system an idealization, and in each case there is some property the system has which the model represents it as not having

[19] A result which Tartaglia also achieved, and for which he cites experimental evidence—see Drake and Drabkin (1969), pp. 64-5.

[20] If we are thinking of a model as a state space with a trajectory running through it, then no specific model of the cannonball's trajectory has been presented—for that, we would need to choose a state space from amongst the various spaces adequate to the job, and specify values of v, α, and m. In the "set of propositions" sense, on the other hand, a particular model has been specified; a more detailed propositional model could simply contain additional propositions concerning the values of the various parameters.

(and, correlatively, some property the system does not have which the model represents it as having). On the regulative proposal I am making, it is correct to talk of idealization with respect to a model of a particular system only when such a state of affairs obtains. To put the rule schematically: If the model represents a system as having the properties $\phi_1, \phi_2,\ldots,\phi_n,\ldots,$ and lacking the properties $\psi_1, \psi_2,\ldots,\psi_n,\ldots,$[21] then a given aspect of the picture of the system presented by the model is an idealization only when that aspect of the picture represents the system as having some ϕ_i which it does not in fact have, and/or as lacking some ψ_i which in fact it has.[22]

There are two things I want to note about this proposal before we go on to formulate a similar proposal concerning abstraction. First, it is important to bear in mind that what matters, according to the proposed necessary condition on being an idealization, is whether, in the relevant respect, the model represents the system as being the way it is; the issue is not whether the model represents the system as being the way *we take it to be*, nor even the way we take it to be when we are speaking as strictly as we can manage. This makes it acceptable to speak of discovering that some assumption made by some model is an idealization, or even of discovering that something we had formerly taken for an idealization is not one (a less likely turn of events, admittedly), and of doing so simply by discovering something new about a certain system. I take it that this comports with much standard usage of the term 'idealization,' and the fact that it does so is a point in favour of the proposal. If, however, it should be decided that some standard usage makes agreement with our Sunday best beliefs the crucial thing, and disregards the question of how the system actually is, or if it should be deemed useful to employ the term in such a way for some philosophical purposes, it will be an easy

[21] The labelling system used here should not be taken too seriously; in particular, I would not wish to assume that either the properties the model represents the system as having, or those it represents the system as lacking, form a countable set.
[22] This may require some modification if we wish to allow for the possibility that it might be indeterminate, in some non-epistemic way or other, whether a given system has a given property, as perhaps some idealizations might then involve a model's ascription of a property to a system, for example, when in fact it is objectively indeterminate whether that system has that property.

matter to introduce a distinction between senses of the term, and some device to make it clear which sense is intended on a given occasion. In this essay, though, calling some aspect of a model an idealization will imply that that aspect distorts the truth of the matter, but will not imply any conflict with the way we take things to be, even if such conflict is often present.

Secondly, it is not clear, and I do not mean to be supposing, that there will be only one way, or even a best way, to individuate the idealizations present in some particular case. For example, the model we have just considered represents the gravitational force of the Earth on the cannonball as constant in both direction and magnitude throughout the region in which the cannonball moves, whereas in fact there will be variation in both respects. Is that one idealization, or two? Or an uncountably infinite number, one (or two) for each spatial point at which the model misrepresents the Earth's gravitational field? There would seem to be little prospect of settling upon a non-arbitrary answer to such questions, and that fact will become important later, when we need to account for talk of degrees of idealization in models.[23] Note, however, that we have here no objection to the coherence of the proposal itself, nothing to prevent us from saying: This model's representing the system as being ϕ_i (or as not being ψ_i) counts as an idealization only if the system is not in fact ϕ_i (or is ψ_i).

Returning now to our model of the flight of the cannonball, we can treat of abstraction in a manner quite parallel to that in which we have just treated of idealization. There are innumerable features of the modeled system which the model omits, without thereby misrepresenting or distorting the system: no mention is made of the composition of the cannonball, of its internal structure, of its color or temperature, of the composition of the ground over which the cannonball flies, of the mechanism by which an initial velocity is imported to the ball, or of the country in which the firing

[23] See section 4.

takes place. The model is simply silent in all these respects. And according to the regulative recommendation I wish to put forward, we should say that a model of a particular system involves an abstraction in a particular respect only when it omits some feature of the modeled system without representing the system as lacking that feature.[24]

The two points just made about my proposal concerning the use of the term 'idealization' apply here, too, *mutatis mutandis*. First, that is, the specified necessary condition for the presence of an abstraction concerns the relationship between the model and the actual features of the modeled system, not the relationship between the model and the features we take the system to have—although, as with idealization, it would be simple enough to recognize another sense of the term 'abstraction' in which it is the latter relationship which is crucial. Second, individuating and counting abstractions would seem necessarily to be a somewhat arbitrary process. Again, this will be important later, when we come to deal with degrees of abstractness (in section 4), but it presents no obstacle to coherently formulating the condition I have just laid out.

The proposals I have just put forward provide only partial guides, for each lays down only a single necessary condition on correct application of the relevant term: misrepresentation for 'idealization,' and omission without misrepresentation for 'abstraction.' We must thus now turn to the twin questions of what more it takes for a misrepresentation to count as an idealization, and what more it takes (if anything) for an omission to count as an abstraction. As we do so, however, the purpose of my remarks becomes a different one; let me take a moment to explain how.

So far, my intention has been to delineate two simple proposals for regimentation of what is currently a confusing tangle of conflicting usages. As I noted earlier, these proposals come into direct conflict with the way some have used the relevant terms;

[24] Talk of a model's "involving an abstraction" is somewhat artificial, but it facilitates a more fine-grained discussion of the various ways in which the model as a whole is abstract, or "is an abstraction"; such phrasing also emphasises the parallels and the contrasts with idealization. Perhaps a more familiar way of capturing essentially the same phenomenon is to speak of various respects in which a given model "abstracts away from" features of the modelled system.

nonetheless, they are quite continuous with the usages of others. We might then hope (and I do) that the proposals in question can also be read as successfully capturing part of the meanings of the terms 'idealization' and 'abstraction' on those usages with which they are continuous. With that hope in mind, we may then be tempted to fall into familiar philosophical habits, and start looking for full accounts of the existing meanings of the terms on those usages by searching for additional individually necessary conditions on correct application, never ceasing until we have a set of conditions which are jointly sufficient. This is not, however, what I intend to do.[25] Instead, my primary aim is simply to identify some factors which are often present when we speak of idealization and abstraction, and which perhaps have something to do with our speaking that way on those occasions when they are present. (Each factor I will mention, incidentally, has also seemed important to at least one other person in their discussion of idealization and abstraction.) No claim is being made that the presence of any of these factors is a necessary condition on the presence of an idealization (or, later, abstraction), nor that their joint presence is a sufficient condition for the same; I leave open, in other words, the possibility that the concepts of idealization and abstraction I am focussing on here are "cluster concepts" of some sort, or perhaps entirely irreducible concepts which are nonetheless importantly connected in some way to the concepts I am about to mention. Whatever the exact semantic structure of things, my hope is that the following discussion (of section 2) will lend greater clarity to our thinking about idealization and abstraction.[26] [27]

[25] The reader should not be misled by the fact that at one point I will consider whether the presence of a certain factor might constitute a further substantive necessary condition on abstraction, and will, at another, ask whether the presence of a certain combination of factors is a sufficient condition for idealization. In both cases I draw a negative conclusion, and my intention, apart from that of generally illuminating the conceptual landscape, is if anything to cast doubt on the project of finding a complete set of individually necessary and jointly sufficient conditions, rather than to engage in it.
[26] I am also not making further recommendations for regimentation in the next section. Should further explicit regimentation be deemed philosophically valuable, on the other hand, I would hope that the following discussion identifies some of the leading candidates for the post of additional necessary condition.
[27] Remember that at this point we are still restricting our focus to talk of idealization and abstraction in models of particular systems, and in specific respects. Other ways of talking about idealization and abstraction in models will be covered in sections 3 and 4.

2. Going further

It is clear that as far as much standard usage is concerned, not just any misrepresentation on the part of a model of a particular system counts as an idealization. One sort of case which makes this clear is that in which the model as a whole is substantially off-target. Consider some specific episode of combustion and a model of the process according to which the burning object releases phlogiston into the air. The model represents the piece of wood, say, as initially containing phlogiston; this is certainly a misrepresentation, but just as certainly it would not ordinarily be called an idealization.

This suggests that perhaps a misrepresentation has to approximate the truth, or appear as part of a model which captures the approximate truth overall, or at least appear as part of a model which gets the basic ontology of the modeled system (i.e., its constituents and central features) right in order to count as an idealization, as opposed to being a useful fiction, say, or an outright mistake. Whether any of these conditions are or should be made necessary conditions on being an idealization is, as I have said, a question I will leave open (although I suspect that the answer is no); but notice that even the conjunction of them, taken in combination with the condition that misrepresentation must be taking place, does not make up a sufficient condition for something's being an idealization. To see this, imagine a classical electrodynamical model I might construct of the flight of an electron though a simple, homogeneous magnetic field, one which gets things almost exactly right except that the mass it attributes to the electron is slightly off—9.108×10^{-31} kg instead of 9.107×10^{-31} kg, say. Here we surely have both misrepresentation and approximate truth in all of the above senses, but it would again seem odd, from the point of view of standard usages, to call the model's attribution of a mass of 9.108×10^{-31} kg to the electron an idealization.[28]

[28] That is, odd to call it an idealization in virtue of those features (misrepresentation with approximate truth) alone. See n. 30.

I will mention just two further features which many idealizations in models of particular systems seem to have, in addition to that of misrepresenting the modeled system in some particular respect. First, as is often noted, idealizations typically make for a simpler model than we would otherwise have on our hands, and they are often introduced for precisely that reason.[29] It is usually presumed that pragmatic concerns drive such maneuvers: the desire to get started, to make a decent prediction relatively quickly rather than a perfect prediction in the distant future or no prediction at all, for example. With simplicity, it is expected, will come tractability. And in the background lies the hope that, should the initial results achieved with the simpler model seem promising, a less idealized and more predictively accurate cousin of the original can be constructed later on, albeit at the cost of greater complexity.[30]

As a distinct issue from simplification, many idealizations can also be said to misrepresent *relevant* features of the modeled system. But relevant to what, and how? Here again pragmatic concerns come to the fore.[31] We often have some specific purpose in mind when we construct a model of a particular system: to explain or make predictions about certain aspects of the behavior of the system, say. (Of course, there may be an ulterior motive driving the pursuit of those goals, such as the confirmation or refutation of a theory.) And that specific purpose might then single out a set of "relevant features" of the system: those which are explanatorily or predictively relevant to these aspects of the behavior of the system. Furthermore, depending on one's views about

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[29] As we saw above (n. 5), Ernan McMullin puts simplification at the heart of his initial characterization of the notion of idealization (which, as he understands it, incorporates both idealization and abstraction in my senses).

Incidentally, it may well be possible to usefully distinguish different kinds of simplicity—mathematical simplicity versus some sort of conceptual simplicity, say, or versus structural simplicity of the system as modelled—but I will not pause to attempt such a thing here.

[30] Thus perhaps the misattribution of a certain mass to an electron, mentioned as an example in the preceding paragraph, might count as an idealization after all if it makes for a model which is somehow simpler, or simpler to use, than it would be otherwise, say by making certain calculations easier (perhaps via some convenient cancelling out which it makes possible).

[31] By 'pragmatic' here, I simply mean "having to do with the purposes for which we have constructed the model in question." One such purpose may be explanation, but no commitment to what is known as a "pragmatic theory of explanation" is implied.

explanation or prediction, one might ultimately take causal relevance, say, or statistical relevance to be the crucial thing.[32] In any case, perhaps we are more likely to call a model's misrepresentation of a system in a given respect an idealization when the misrepresented feature (or features) of the system is (or are) taken to be relevant, in the appropriate manner, to those aspects of the behavior of the system which we were primarily concerned to predict or explain when we constructed the model. (Certainly we are more likely to want to draw attention to misrepresentations of features which we take to be relevant to the behavior in which we are especially interested.) And, independently of whether that is so, it might prove fruitful in certain contexts to regiment our terminology in such a way that idealizations necessarily misrepresent relevant features.[33] [34]

We turn now from idealization to abstraction, and ask what more there is to abstraction than omission. Having just discussed three important things which some, and perhaps many idealizations do—approximating the truth, contributing to simplicity, and misrepresenting relevant features of the modeled system—it is natural to wonder whether there is a parallel story to be told about abstractions.

The first and quite trivial point in this regard is that clearly there is no sense in which an abstraction can approximate the truth, nor any interesting sense in which it can fail to, simply because when a model contains an abstraction with respect to some particular property, it is entirely silent on the matter of whether the system it models has

[32] And of course one might believe *those* sorts of relevance to be intimately related in one way or another.

[33] Note that if we do think of idealizations as involving the distortion of relevant features, and if relevance is elaborated along the lines I have sketched here, then which features of a given model are correctly called idealizations might vary with the pragmatic context of use of the model—what is and is not an idealization, in other words, may depend in part upon what we are at a given moment hoping to predict, or explain. On the other hand, it may be that certain elements of the model misrepresent features of the system which count as relevant for most of our typical purposes, in which case we may classify those elements of the model as idealizations without any particular context in mind.

[34] On the account Nancy Cartwright offers (1989, chapter 5; see esp. p. 187 and pp. 190-1), both simplification and relevance play a crucial role. According to Cartwright, the point of constructing an idealized model (that is, a model containing a number of idealizations) is to single out the causal capacities of one particular factor in a given situation, and such a model does that by representing all other potentially causally relevant factors as absent. In doing so, the model typically both simplifies the causal structure of the situation, and misrepresents how things stand with respect to a number of causally relevant features of the system in question (by representing such features as absent when they are present).

the property or lacks it; nor is it obvious (to me, anyway) that there is any other closely related job which abstractions can do.

Might a contribution to overall simplicity be a crucial condition on an omission's being an abstraction, or at any rate on being an interesting abstraction? Well, simplification would seem to be an automatic consequence of omission, in that, of two models of a given system, the one which omits mention of a certain feature of the system will thereby be the simpler model, *ceteris paribus*. Thus, although it might be true that abstraction always contributes to simplification, it would be of no help to say that an omission counts as an abstraction only if it contributes to the simplicity of the model—all omissions do. Still, it is worth noting that abstractions always contribute to simplicity, as this is no doubt one of the reasons we employ them.

With regard to the role of relevance, matters are perhaps a little less straightforward. Both Nancy Cartwright and Ernan McMullin seem committed to the claim that models will contain abstractions (in my sense) only with respect to features which we deem irrelevant, for they each suggest that in constructing a model we will always include *some* assumption about the presence or absence of any feature of the modeled system which we deem relevant to the behavior we seek to explain or predict (Cartwright (1989, p. 187) and McMullin (1985, pp. 258-64, esp. p. 258)).[35] This seems plausible enough, especially if we are sufficiently liberal on the question of what assumptions a given model should be taken to include. For example, in constructing a model of the cannonball firing discussed above, we might not expressly postulate the

[35] McMullin distinguishes between two central sorts of idealization (which, the reader will recall, encompasses both idealization and abstraction in my sense of those terms) in models, calling them "formal" and "material" idealization, respectively (1985, pp. 258-64). Formal idealizations deal with relevant features of the modelled system, and material idealizations with irrelevant ones. Although McMullin does sometimes write of formal idealizations which "omit" features of the modelled system, this always seems to mean "omit by representing as absent," and so in context to imply misrepresentation, as opposed to meaning "omit by not mentioning," which is the sort of omission that abstraction in my sense must involve. It is McMullin's material idealization, dealing as it does only with features which are deemed irrelevant, which involves omission in the sense I have made crucial. (More carefully we might say "not deemed relevant," as in some cases the feature is one we have not conceived of at the time the model is constructed—consider McMullin's own example of electron spin, discussed on pp. 263-4).

absence of air resistance, but we might nonetheless be said to have constructed a model in which it is assumed that there is no air resistance, simply because it is Earth's gravitational force on the cannonball which we feed into Newton's second law, a law which relates mass and acceleration to the *total* force on the body.

Note, however, that there are still two reasons for saying that models can abstract away from relevant features of the modeled system. First, even to the extent that Cartwright and McMullin are correct, there may be relevant features of the system which we mistakenly deem irrelevant, and abstract away from when constructing our model. Secondly, models sometimes seem to abstract away from features which we in fact deem relevant, but to make some distinct idealizing assumption which "screens off" the presence or absence of the relevant feature.[36]

This latter point is nicely illustrated by a case McMullin discusses, in fact. To derive the ideal gas law for some specific body of gas using the kinetic theory of gases, we construct a model of the gas according to which the molecules making it up are perfectly elastic spheres which exert no attractive forces upon one another (1985, p. 259). As McMullin emphasizes (*ibid*., p. 258), such a "billiard ball" model of the gas makes no mention of the internal structure of the molecules. Yet that internal structure is surely relevant to predicting the way in which the pressure, volume, and temperature of the gas will covary, for it is the internal arrangement of the parts of the molecules and of the atoms which make them up that gives rise to the attractive intermolecular Van der Waals forces, and once those forces are taken into consideration, a different equation relating pressure, volume, and temperature results. In the original model, the possibility that such relevant features of the modeled system have been ignored is "covered," so to speak, by the idealization that there are no attractive intermolecular forces; nonetheless, it seems accurate to say that the model omits mention of a feature of the modeled

[36] I am using the term 'screens off' in a general figurative sense here, and not in its technical probabilistic sense.

system—namely, the structure of its molecular components—which is relevant to the behavior with which the model is concerned.

Thus models do sometimes abstract away from relevant features of the systems they model; and when they do so, that is an important fact about them. If we wished to stress the importance of relevance, we might choose to stipulate that abstraction should only be spoken of when what is omitted is a relevant feature of the system. We might then want to consider a related idea suggested by (although not explicitly contained in) work of Cartwright and Mendell (1984) and Griesemer (this volume), that an abstraction properly so-called should always involve the omission of a feature of the modeled system which is of one explanatory kind or another.[37] Alternatively, we could take the more generous line that although there is nothing more to abstraction *per se* than omission (omission without misrepresentation, that is), the omission must be of a relevant feature, in the sense indicated above, if it is to be an interesting or important abstraction. I shall not, however, try to weigh the relative merits of these two approaches here, nor even try to determine whether there is anything at stake in the choice between them.

This, then, concludes my delineation of the two notions I am taking as basic: the notion of a model of a particular system idealizing (or containing an idealization of) the system in a given respect, and the notion of a model of a particular system abstracting away from (or involving an abstraction with regard to) some specific feature of the system. In the next two sections, I hope to showing how we can extend the account

[37] See also Cartwright (1989, pp. 212-24). For Cartwright and Mendell the taxonomy of explanatory kinds is derived from Aristotle's four-fold taxonomy of causes; for Griesemer it is provided by background scientific theories. These authors have a different quarry in mind than I do at this point: rather than trying to say when a feature of a model counts as an abstraction, or what it means to say that a model abstracts away from the concrete situation in this or that respect, they are concerned to provide a way of ordering models, theories, and the like with regard to their overall degree of abstractness. Note, however, that there is nonetheless an internal tension in Cartwright's views here: if a representation is more abstract when it specifies features of fewer explanatory kinds, then if there are to be different degrees of abstractness, there will have to be representations which abstract away from features which are at least explanatorily relevant, in which case it is hard to see how it can be true that "a model must say something, albeit something idealizing, about all the factors which are relevant" (*ibid*., p. 187).

developed thus far to other, related ways of talking about idealization and abstraction in models, and so illuminate them.

3. Models of kinds

It is not too difficult to see how we might extend the ideas discussed above to the case of a model which is, or is on some occasion functioning as, a model of a kind of system, rather than a particular system. The model will represent things of kind K, the κ's (neutrons, carbon atoms, cells, free-market economies, ecosystems...), as each having properties ϕ_1, ϕ_2, …, ϕ_n, …, and as failing to have properties ψ_1, ψ_2, …, ψ_n, …; it will also omit any mention of a very large number of properties χ_1, χ_2, …, χ_n, …. In the spirit of the regulative proposals described above, then, we can adopt the rule that the model should be said to contain an idealization in representing the κ's as having ϕ_i only if some κ's fail to have ϕ_i, and that the model should be said to contain an idealization in representing the κ's as failing to have ψ_i only if some of the κ's have ψ_i.[38] Similarly, we can stipulate that a model of a kind K should be said to contain an abstraction with respect to the property χ_i (of which it makes no mention) only if some κ's have χ_i.[39]

In addition to imposing this bit of regimentation, we then add that idealizations often approximate the truth about those κ's they misrepresent (or at least most of them), that idealizations can contribute to the degree of simplicity the model enjoys, and that idealizations and abstractions are at their most interesting when they distort the truth about, or (respectively) omit mention of features which are relevant, in some specified

[38] Note that 'some κ's' can strictly be read as meaning "at least one κ"; it just that if the model misrepresents only one κ in the relevant respect, then we are likely to regard the model as idealizing the κ's only very slightly. See the account of talk of degrees of idealization in section 4.

[39] In some cases we speak of a model's abstracting away from a quantity (understood as a determinable), such as the electric dipole moment of the molecules in an a gas, or a qualitative determinable, such as the colour of a cannonball. A model of a kind K should then be said to contain an abstraction with respect to a certain determinable (qualitative or quantitative) when it makes no mention of that determinable (nor any of the corresponding determinate properties), but some of the κ's have one determinate property or other from the set of determinate properties corresponding to the determinable. (Typically, I suppose, if one of the κ's possesses a determinate property from the set corresponding to the determinable, then they all will, but there may be exceptions to this.) A similar point applies to the notion of abstraction in a model of particular system.

sense, to the behavior we are concerned to study. It is perhaps also interesting to note that idealizations in models of kinds sometimes ascribe a property, ϕ_i, to the κ's which is in one sense or another a limiting case of some range of properties actually instantiated by the various κ's. (For example, ϕ_i might be the property of experiencing zero air resistance, in a model of pendula.)[40]

4. Degrees of idealization and abstraction

Until now, whether dealing with models of kinds or models of particular systems, we have been focussing on uses of the term 'idealization' and 'abstraction' on which idealizations and abstractions are tied to particular features of systems, the particular features they misrepresent or omit to mention, respectively. But models themselves are sometimes said to be idealizations, or (more commonly) to be idealized, without any further reference to specific respects; one model can be said to be more idealized than another; and the term 'idealization' can also be used to denote a process which occurs as part of scientific work. The same holds, *mutatis mutandis*, for the terms 'abstraction' and 'abstract.' Can we employ the account given above of the first sort of locution as the basis of an account of the others? I will consider this question first with respect to the various sorts of talk of idealization; the case of abstraction is parallel up to a point, but there is an important difference, and I will describe that difference after dealing with idealization.

On some occasions, when we say that a model M is an idealization, or is idealized, we mean just that it contains one or more idealizations in the sense discussed

[40] One difficulty for this approach to understanding talk of idealization and abstraction in models of kinds is that it clearly cannot be extended straightforwardly to talk of idealization and abstraction in models of uninstantiated kinds, relying as it does on there being some κ's to have, or fail to have, one or another property. Perhaps when a model of an uninstantiated kind (such as a relativistic model of some particular kind of universe very unlike our own) is said to contain an idealization or an abstraction, it is implicitly being thought of as a highly idealized model of an actual kind or particular (such as the actual universe). Or perhaps we might appeal to modal facts about what properties κ's would have were there any. (This latter approach seems more promising than the former when dealing, say, with a model of some sort of molecule which does not occur naturally and which we have never synthesized.) These are topics for further investigation.

above (in sections 1 and 2); and talk of idealization as a process or activity may simply denote the process or activity of constructing an idealized model in this simple sense. An account of such talk can thus be derived straightforwardly from the initial account of idealizations as aspects of models.

Matters become significantly more complex, however, when in calling *M* an idealized model we mean to say, not just that it contains at least one idealization, but that it is *highly* idealized, or when we explicitly say just that; when we say that *M* is a more, or less, idealized model than some other model; when we speak of a process or activity of idealization and mean the process or activity of constructing a series (two-membered, in the trivial case) of increasingly idealized models; or when we speak of "de-idealization" or "correction," and mean the converse process of constructing a series of increasingly *less* idealized models. (A term often used to describe the parallel activity of constructing less and less abstract models is "concretization."[41] We also speak of one model's being more "realistic" than another, and it seems to me that this can mean that the more realistic model is less idealized, or that it is less abstract, or both.) What these ways of talking have in common, obviously, is that implicitly or explicitly, they all express judgements (often comparative judgements) about the degrees of idealization various models exhibit. In making such judgements, I want to suggest, we are typically taking into account a number of different factors.

Consider first a model, *M*, of a particular system. The natural approach here is to regard the question "How idealized a model is *M*?" as made up of two component questions: (i) How many idealizations does *M* contain? (ii) How much of an idealization is each of the idealizations which *M* contains? The answer to the original question, about the overall degree of idealization *M* enjoys, can then be arrived at by taking a sort of

[41] Juan Gris's description of the relationship between his work and Cézanne's provides a nice parallel to the discussion of abstraction and concretization as converse processes of model construction in science: "I try to make concrete that which is abstract…Cézanne turns a bottle into a cylinder, but I make a bottle—a particular bottle—out of a cylinder" (quoted in the entry on Gris in Chilvers (1990)).

weighted sum over all the particular idealizations *M* contains, in which attaching a heavy weighting to a particular idealization amounts to claiming that that idealization idealizes the modeled system to a significant degree, and so on.[42]

This little account of the overall extent to which a model of a particular system is idealized begs a question, however, because it assumes that we already have a well-defined way of talking about the degree to which a *particular* idealization, contained within the model, idealizes the system in question.[43] So how is this latter quantity measured? It seems to me that all we mean to do when we talk about the degree to which model *M* idealizes system *S* by representing it as having property ϕ_i (as when we exclaim, for example, "That's a *huge* idealization!") is to single out the degree to which *M* misrepresents, or distorts the truth about *S* in representing it as being ϕ_i.

This claim may seem unenlightening, but the point of it is to be found in what it denies. To see this, recall that we discussed three things idealizations sometimes do, in addition to simply misrepresenting the way the system is: approximating the truth, simplifying the model, and misrepresenting a feature of the system which is relevant to explaining or predicting its behavior, say (or relevant in some other way which is determined by the aims we have in constructing the model). The point then is that although simplification and relevance both come in degrees, at least some of the time, I am denying that when we size up the degree to which a particular idealization idealizes, degrees of simplification or relevance are any part of it. (Sometimes small idealizations can simplify greatly, and sometimes they can misrepresent highly relevant features.[44])

[42] This talk of weighted sums is not to be taken too seriously: I do not mean to suggest that in making judgements about the degree of idealization a model enjoys we employ some precise algorithm, nor that we are always, or even usually able to attach precise strengths to the various idealizations the model contains, nor even that we are interested in doing so.

[43] The account begs another question, too—about how are to count idealizations. This difficulty was foreshadowed in section 1 (see the text containing n. 23), and I will returnto consider it at the end of this section.

[44] Not that these claims are strictly incompatible with the idea that degrees of simplification and of relevance are part of what we are evaluating when we evaluate the degree to which a particular idealization idealizes: we might be taking a weighted sum of the degree of distortion, the degree of simplification, and the degree of relevance, and simply weighting the degree of distortion significantly more heavily than either of the other two factors. Nonetheless, it seems to me as a matter of linguistic intuition that that is not how we do it,

All that matters, when we evaluate the degree of a particular idealization, is how far from the truth it is for *M* to represent *S* as ϕ_i. This does mean, on the other hand, that when an idealization approximates the truth, the degree to which it does so does bear on the question of how much the idealization idealizes, for clearly a higher degree of approximation means a lower degree of distortion or misrepresentation, and *vice versa*.[45]

A certain nervousness may arise at this point about the notion of degrees of misrepresentation or distortion, provoked perhaps by the apparent link to the notion of approximate truth, a notion with a troubled history in the philosophy of science.[46] In this case, however, it seems to me that the failure of philosophical attempts to provide a precise account of the notion of approximate truth cannot mean that there *is* no coherent notion by that name, nor that we cannot meaningfully speak of the degree of misrepresentation or distortion present in a given aspect of a given model. There is clearly sense (and, indeed, truth) in saying that a model which represents a certain smooth metal ball rolling down a certain icy slope as experiencing no frictional force approximates the truth, if in fact there is just a small amount of friction present. We can just as clearly compare degrees of misrepresentation in specific respects, in at least some cases: we can draw such a comparison, for example, between the model of a cannonball firing presented above (in section 1) with a model of that same event which takes into account gravitational forces on the cannonball due to the moon. And comparing degrees of idealization in specific respects in models of *distinct* systems can also be a quite

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and the alternative picture I am sketching accounts for the facts stated in parentheses in the text far more straightforwardly.

[45] This does not necessarily mean that the question of the degree of misrepresentation and that of the degree of approximate truth involved in an idealization are interchangeable; whether they are depends on how we understand the notion of approximate truth. Crudely put, the telling question in this regard is: Does a representation have to be approximately true to have any degree of approximate truth? To put the point slightly less crudely: If we understand the notion of approximate truth in such a way that it is possible for partial truths to lack any degree of approximate truth (sc., partial truths which are sufficiently close to being complete falsehoods), then we may want to countenance degrees of distortion which distinguish between representations all of which have the same degree of approximate truth, namely, zero. If, on the other hand, any partial truth, no matter how partial, is to have some degree of approximate truth, then perhaps degrees of misrepresentation and degrees of approximate truth may be regarded as interdefinable.

[46] See, for example, Niiniluoto (1998).

straightforward matter: If we were to fire a cannonball on an open plain on the moon, and model the system along the same lines as those sketched earlier, taking into account only gravitational forces on the projectile due to the body on which the episode occurred, then we could straightforwardly say that the resulting model would misrepresent the gravitational forces acting on the cannonball to a greater degree than that to which our original model idealizes the gravitational forces acting on its cannonball, as the moon model would be laying aside the Earth's gravitational influence on the projectile whose motion it represents, and this is in a simple quantitative sense a greater distortion than the one in which the Earth model indulges when it eliminates the moon's influence on the earthbound cannonball.[47]

In the cases just described, a precise quantitative measure of degrees of distortion in specific respects can simply be read off from the precise measures of the quantities modeled (namely, frictional and gravitational forces).[48] Meaningful talk of degrees of distortion also seems intuitively possible in some cases where we have no ready way of measuring with any exactness the feature of the system misrepresented in the model: returning to our early example from Chomsky, some speakers have more limited memories, and are more easily distracted than others.[49] This is not to say that fine-grained discriminations can always be made, however—often attempts to compare degrees of idealization in specific respects will lead only to a partial ordering, and not due to some limitation in our powers of discernment. And, more importantly, I should

[47] I am assuming here that this difference is not compensated for by other differences in the gravitational forces ignored by the two models, differences which will be due to relatively small differences in the distances of various heavenly bodies from the Earth and from the moon. In any case, the judicious placement of a black hole can provide us with a more clear cut example, if one is needed.

[48] This talk of idealizing in respects and to degrees echoes Ronald Giere's language in his (1988). According to the view of theory structure he presents there, a "theoretical hypothesis" makes claims about the respects in and degrees to which a certain model is similar to a particular system, or type of system, and, roughly speaking, we might say that there is an inverse relation between degree of similarity and degree of idealization. For criticisms of Giere's notion of a model and his talk of similarity, however, see Jones (forthcoming b).

[49] This is not to say that precise measures of, say, memory capacity could not be constructed—indeed, psychologists have constructed such measures. The conceptual point is made, however, as soon as we recognise that those of us who have at our disposal no such precise measures can clearly make judgements of degree on these scores nonetheless, and thus could judge one given model to be a more idealized model of a certain speaker than another, at least in one of these particular respects.

emphasize that in recognizing talk of degrees of misrepresentation, of distortion, and of truth as legitimate in general, I am not assuming that such talk is always meaningful. Perhaps in some cases it simply does not make sense to ask how close to the truth 'S is ϕ' is, or how great a misrepresentation it is. On the account given here, this simply implies that it will not be possible to talk about the degree of idealization involved in this aspect of the model. The account offered, such as it is, is intended only as account of the content of judgements when they can be made.[50]

We are now in a position to outline an understanding of talk of degrees of idealization in models of kinds. At the lowest level of resolution, the natural approach is exactly parallel to the one we adopted for models of particular systems: We break the question "How idealized a model is *M*?" down into the two questions (i) "How many idealizations does *M* contain?" and (ii) "How much of an idealization is each of the idealizations which *M* contains?", and find the answer to the original question by taking a sort of weighted sum over the various particular idealizations contained in *M*, bigger weights being assigned to bigger (particular) idealizations. The difference in this case, however, lies with the fact that question (ii) can itself then be divided in two. We begin by classifying *M*'s ascription of ϕ_i to the κ's (systems of the kind K) as an idealization just in case there is at least one κ which lacks ϕ_i. Then for each particular idealization *M* contains we must ask (iia) "What fraction of the κ's does *M* idealize in this respect?"; in addition, for each κ it idealizes, we must then ask (iib) "To what degree does *M* idealize κ_j by representing it as being ϕ_i?" (quite possibly receiving different answers for different κ's). Thus, to put it another way, when dealing with models of kinds, the answer to question (ii) is itself arrived at by taking, for each idealization the model contains, a weighted sum over all the κ's which that idealization idealizes, bigger weights corresponding to greater degrees of idealization in the sense discussed above (i.e.,

[50] Note that nothing I have said prevents the picturing of S as ϕ from counting as an idealization *simpliciter* in such a case.

greater degrees of misrepresentation or distortion), and then dividing by the total number of κ's (to get a fraction).[51] Putting it more intuitively, we can say that the degree to which a particular idealization idealizes the kind is a combination of two factors, namely, what fraction of things of that kind it idealizes, and how much it idealizes the ones it idealizes.[52]

Extending what I have been calling "the natural approach" to talk of degrees of abstraction (or abstractness) results in a considerably simpler story than the one I have just told about degrees of idealization, and for two reasons. First, there is no question of evaluating the degree to which a model omits a given feature of a given system, and so no parallel to the talk of degrees of misrepresentation or distortion we employed in the case of idealization. This fact then seems to preclude talk of the degree to which a model abstracts away from a feature of a particular system. Second, in the case of a model of a kind, it would seem that if a model abstracts away from a given feature possessed by one system of that kind, then it abstracts away from that feature, or from other determinate of the same determinable, for every system of the kind. So, taking color as our example of a determinable, if a model of pendula abstracts away from the redness of this pendulum, then it will abstract away from the redness, or blueness, or greenness, as the case may be, of any other pendulum. And counting the fact that a model abstracts away from the redness of red pendula and the fact that it abstracts away from the greenness of green pendula as two separate abstractions would seem to be double counting. The point might be put by saying that, when counting abstractions, we should count the number of determinables the determinates of which a given model abstracts away from. And a model cannot abstract from the determinates of a certain determinable in the case of some systems but not in the case of others. Thus there is no

[51] Again, this mathematical talk should not be taken too literally; see n. 42.

[52] There may be a difficulty here with the idea that what is relevant is the make-up of the class of actual κ's. Intuitively, the worry would be that the actual κ's may not be representative of the kind, especially if by 'actual κ's' we mean the κ's existing at some particular time. For some preliminary thoughts on a related problem, see n. 40.

analogue, when interpreting talk of degrees of abstraction with respect to a model of a kind, to counting the number of systems of that kind which the model idealizes in some given respect. Given these two points, then, in the case of abstraction the natural approach thus reduces to the idea that to ask about degrees of abstraction is just to ask how many abstractions the model contains, both in the case of a model of a particular system and in the case of models of kinds.

An intuitively appealing way of making sense of talk about overall degrees of idealization and abstraction in models readily suggests itself, then. And given such an account, we seem to be in a position to make equally good sense of the various sorts of talk about idealization and abstraction listed at the beginning of this section. Unfortunately, however, there is a serious problem which threatens to undermine all this. It is a problem which has its roots in something we noted on first introducing the idea of a model's idealizing or abstracting away from the features of a system in some particular respect (in section 1), namely, that there is typically no way of individuating the idealizations and abstractions contained in a model which is obviously to be preferred. There is thus, in general, no straightforward way of saying how many idealizations a given model contains, nor how many abstractions. Consequently, given the details of the approach to judgements of overall degrees of idealization and abstraction outlined in this section, it becomes quite unclear how such judgements could ever be nonarbitrary. Yet (and this is the other horn of our dilemma), if judgements concerning overall degrees of idealization and abstraction are not understood to be, in part, judgements about the sheer numbers of idealizations and abstractions various models contain, it is not clear how they are to be understood.

Admittedly, there may be ways around this difficulty with regards to comparative judgements in certain special circumstances. In certain cases it may be clear that, however we count the number of idealizations (or abstractions) in each of two models, M and M', those contained in M are a proper subset of those contained in M'.

(M' might have been obtained from M by means of a single modification, or *vice versa*.) There is perhaps some hope that adjacent members of the series of models alluded to in talk of idealization and correction as processes (or of similar talk of abstraction and concretization) will be related in this way. In such cases, at any rate, we clearly can make sense of comparative judgements of degree of idealization (or abstraction) in a way which is entirely parasitic on our basic account of what it is for a model to contain an idealization (or abstraction) in a specific respect. If it is true, however, that some of our talk about idealization and abstraction presupposes an ability to make coherent judgements of degree (some of them comparative) in cases where this simple "proper subset" relation does not hold, then we have an unresolved problem on our hands.

As mentioned above, Cartwright and Mendell (1984) propose an account of the content of judgements of degrees of abstractness which relies on a taxonomy of explanatory kinds akin to Aristotle's, and Griesemer (this volume) advocates an importantly modified version of their account.[53] It is thus interesting to investigate whether either of those accounts succeeds, and whether such an approach can be extended to the case of idealization. It is worth noting, in any case, that Cartwright and Mendell's and Griesemer's accounts, when applied to models, rely on the prior notion of an abstraction as a particular feature of a model.[54]

5. Laws: Preliminaries

So much for the discussion of how we should understand talk of idealization and abstraction as it applies to models, and how we might usefully regiment such talk. It is now time to think about laws.

The notion of law is itself hardly an unproblematic one, of course. Debates about the logical form of law statements, the precise role of laws in theories, the objectivity of

[53] Again, see also Cartwright (1989), pp. 212-224.
[54] Although not in those words. Cartwright, for example (*op. cit.*, p. 220), writes of those features which are and are not "specified."

lawhood, and the very possibility of drawing a principled distinction between laws and accidental generalizations, so-called, are as unresolved as they are familiar. Despite the existence of radical disagreements over such matters, however, few would dissent from the rough formula that statements intended to express scientific laws, at least for the most part, take the form[55]

$$\phi\text{'s are } \chi\text{'s.} \qquad (N)$$

(where '*N*' is for 'nomological'). The claim I intend to make here concerning statements of law is quite minimal. It leaves open, for example, the question of whether the logical form of such statements is adequately captured by sentences of first-order predicate calculus of the '$(\forall x)(\phi x \rightarrow \chi x)$' variety,[56] whether they instead assert that a certain pair of universals, ϕ–ness and χ–ness, stand in a relation of necessitation to one another,[57] or whether they should be understood primarily as making a claim about the capacities ϕ's have in virtue of being ϕ's.[58] I also do not assume that English statements of the form

All ϕ's are χ's

provide accurate paraphrases of statements of form (*N*), or even that the law statement 'ϕ's are χ's' entails a claim which has the logical form '$(\forall x)(\phi x \rightarrow \chi x)$.' One might, for example, regard some or all law statements of form (*N*) as accurately paraphrased by statements of the form

ϕ's tend to be χ's,

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[55] Or some closely related form, such as 'ϕ's are followed by χ's,' 'ϕ's have χ,' etc. (where 'ϕ' and 'χ' are doing multiple duty as stand-ins for various parts of English). For the purposes of brevity, I will take 'ϕ's are χ's' to be canonical. 'ϕ' and 'χ,' of course, can stand for pieces of scientific English which are syntactically far more unwieldy than the Greek letters themselves (and the use of standard examples concerning black ravens and white swans) might suggest.
[56] At least for the most part—see the discussions of the "contrapositives objection" in sections 7 and 8.
[57] See Dretske (1977), Tooley (1977), and Armstrong (1983).
[58] See Cartwright (1989), esp. chapter 5, for such a view.

reading this paraphrase in such a way that it entails 'Most ϕ's are χ's,' or 'Many ϕ's are χ's,' but not 'All ϕ's are χ's.' Or one might take a law statement of form (*N*) to be more accurately paraphrased by a statement of the form

All ϕ's, in virtue of being ϕ's, have the capacity to (be) χ,

the truth of which is, I take it, quite compatible with there being no ϕ's which are χ, even if there are ϕ's.[59]

Attributing form (*N*) to law statements is thus intended to commit us to very little; nonetheless, it provides us with a sufficient foothold to allow us to make some substantive claims about idealization and abstraction in laws, whilst at the same time allowing us to remain neutral on the standard philosophical issues just mentioned.[60]

Before outlining the remainder of what I have to say about laws it will be helpful if we first fix a piece of terminology: I will say that the law that ϕ's are χ's, or a law statement (which may or may not express a genuine law) of the form 'ϕ's are χ's' *applies* to a given system just in case the system is a ϕ.[61]

The discussion of idealization and abstraction in laws, then, will proceed as follows: I will begin by devoting some time to distinguishing and characterizing three different sorts of law-related idealization. First there are statements which are of form (*N*) which we treat as statements of law for some purposes, and which may apply to a

[59] I emphasise these last two readings with Cartwright (1989) in mind. In her terms (inspired by Mill), the former reading provides us with a "tendency law," the latter with a "law about tendencies" (p. 178). See also chapter 5, section 5 of her book.

Some might say that a statement cannot express a "genuine" law, or a law in some especially important sense, if it does not entail the corresponding '$(\forall x)(\phi x \rightarrow \chi x)$' statement. If that is so, however, then we may need to reckon with the possibility that the statements which we typically classify as expressions of law in actual scientific practice do not express "genuine" laws, or laws in the especially important sense. Be that as it may, we are concerned here with understanding talk of idealization and abstraction as it applies to actual scientific practice, and the things we classify as laws whilst engaged in that practice.

[60] Note that if there are laws which do not take form (*N*), then what I have to say may not cover them. On the other hand, it seems to me that it would be a relatively straightforward matter to extend the following account beyond the domain of laws to general scientific claims of all sorts, provided only that they have the right sort of form.

[61] There is a certain worry one might have here about whether the notion of application thus defined is well-formed in the case of laws (rather than law statements). For a discussion of that point, see the discussion which closes section 7 below.

large number of systems, but which are actually false, and false in a way which makes the statements themselves idealizations. For convenience, I will say that such statements express "quasi-laws." Secondly, there are genuine laws typical employment of which nonetheless involves idealization; here the idealization is required in order that we may regard the law as applying. I will refer to such laws as "idealized laws." And thirdly, there are genuine laws which only truly apply to systems which are ideal in some sense; these are the "ideal laws."[62] The distinction between the second and third sorts of law-related idealization may be especially unclear at this point, but things will become clearer below.[63] My elaboration of the various distinctions will rely very centrally on that fundamental notion an account of which provided the starting point for the account of idealization in models given in the sections 1 and 2—that is, the notion of idealizing a particular system in some specific respect. Briefly, the idea is (in part) that when a law statement of the form 'ϕ's are χ's' expresses a quasi-law, then representing the systems we are dealing with as χ's is an idealization in at least some cases, whereas when it expresses either an idealized law or an ideal law, our use of the law requires us in many, most, or all cases to indulge the idealization that various systems are ϕ's.[64] Following all of this, I will then provide a relatively quick account of abstraction in laws, one which builds in a simple way on the account of abstraction in models presented above.

6. Quasi-laws

Unfortunately, the precise articulation of the notion of a quasi-law which best captures the intuitive idea will depend on the account of law statements to which one subscribes. If a statement of form (*N*) is taken to be, or to entail a statement of the form '$(\forall x)(\phi x \rightarrow \chi x)$,' then given that being a quasi-law is a matter of its being an idealization to represent

[62] For brevity's sake, I will also apply these labels to the statements which express such laws, whenever doing so will produce no confusion.
[63] It may help to bear in mind that the categories of idealized law and of ideal law are (at least) overlapping. Again, see below.
[64] Although it will turn out that this is only contingently true of ideal laws.

the ϕ's as χ's, it will be a necessary condition on some statement's being a quasi-law that not all ϕ's are χ's. This necessary condition must be replaced by a stronger one if an alternative account of laws is adopted: If the emphasis is on tendency laws, which say only that most or many ϕ's are χ's, then we have a quasi-law only when it is *not* the case that most or many ϕ's are χ's; and if a law statement of form (*N*) is to be paraphrased as 'All ϕ's have the capacity to χ,' then for the statement to express a quasi-law it must be the case that some ϕ's do not have the capacity to χ. In each case, it follows that the statement expressing a quasi-law is false, and that is a crucial part of the notion I am attempting to characterize.[65] For ease of exposition in other parts of this discussion, however, and without meaning to prejudice the issue, I will assume hereafter that we have adopted an account of laws on which a law does entail the corresponding '$(\forall x)(\phi x \rightarrow \chi x)$' statement. I think it will be clear how to modify the following remarks in order to accommodate views on which this entailment does not hold.

A necessary condition, then, for a statement of form (*N*) to express a quasi-law, given the assumption I have just made, is that some ϕ's are not χ's (regardless of what we believe in that regard).[66] Another is that, at least for some purposes, we treat the statement as a law, citing it in explanations, employing it to make predictions, using it to support counterfactuals, and so on. Even together these conditions are clearly not

[65] The list of sample necessary conditions is included because it is not quite enough to say "Law statement *L* must be false to express a quasi-law"; *L* must be false for the right reasons, so to speak. If, for example, law statements should be understood as claiming to express relations of necessitation between universals, then it is not enough for no such relation of necessitation to hold between the universals in question—it may still be accidentally true that all the ϕ's are in fact χ's, in which case I see no reason to speak of idealization (as opposed to error of another kind). The crucial feature of a quasi-law I am trying to get at might be rather imperfectly put this way: If *L* is a quasi-law, then the "extensional content" of *L* must be false. And for an illustration of what I mean by this imprecise use of the phrase 'extensional content,' I must then refer the reader back to the list of sample necessary conditions provided in the text.

[66] A difficulty arises here in the case in which there *are* no ϕ's, as intuitively it seems that a statement of the form 'ϕ's are χ's' might still be treated as a law in some circumstances, and might nonetheless be an idealization in something like the way I am trying to characterize. (Note that this is reminiscent of some problems we encountered in understanding talk of idealization with respect to models of uninstantiated kinds.) Perhaps we might attempt to address this difficulty by invoking counterfactuals, and making it a necessary condition on such a statements's being a quasi-law that if there were ϕ's, not all of them would be χ's. It is easy to see discover problems with at least the initial statement of this solution, however; whether those problems could be overcome, or whether the original difficulty can be dealt with in some other way, will be left as a question for further exploration.

sufficient, however, if being a quasi-law is to involve idealization in some way: some statements of form (*N*) which at one time or another we have treated as expressing laws for various purposes (and which we have perhaps taken to express actual laws) have simply been false, without being or involving idealizations, and without it being appropriate to say that we were idealizing in treating the statements in question as expressing laws. So what else is involved in something's being a quasi-law?

Recalling the discussion of idealization in models, we can immediately identify three important features a quasi-law may have. One is approximation: 'ϕ's are χ's' might be approximately true. Another is simplicity: 'ϕ's are χ's' might be a simplification of the truth about ϕ's. And finally there is relevance: saying that ϕ's are χ's might misrepresent features of the non-χ ϕ's which are relevant in one or more ways to the purposes we have in mind when employing the quasi-law as though it were a genuine law.[67] Given this, we can settle on the following characterization of the notion of a quasi-law:

> *L*, a statement of form 'ϕ's are χ's', is (or expresses) a *quasi-law* if and only if (i) *L* is treated as a law for some purposes, but (ii) some of the ϕ's are such that it is an idealization to represent those ϕ's as χ's.

It follows, of course, from condition (ii) and my account of particular idealizations in specific respects (in sections 1 and 2) that some of the ϕ's are not χ's if *L* states a quasi-law, and thus that statements of quasi-laws are false, and (so) do not express laws.[68]

To illustrate the notion of a quasi-law, consider as a simple example the law of gravitational free fall. A typical statement of this (so-called) law might run as follows:

[67] This, however, seems virtually guaranteed, and to the extent that it is, relevance so elaborated seems ill-suited to be an independent criterion of quasi-lawhood. Perhaps instead we might focus on the distinct question of whether representing the non–χ ϕ's as χ's misrepresents them in a way which is relevant to our purposes—whether, for example, this misrepresentation makes a difference to the predictions are interested in making. The distinction here is between merely misrepresenting relevant features, and misrepresenting relevant features in a way that makes a (relevant) difference.

[68] My intentions here are quite parallel to my intentions in the discussion of section 2, as described at the very end of section 1. Although I do provide a pair of individually necessary and jointly sufficient conditions for correct application of label 'quasi-law', so that there is a surface-level difference, the second of those conditions employs the notion of a particular idealization in a specific respect, a central notion for which I have not tried to give necessary and sufficient conditions.

Near the surface of the Earth, a falling body accelerates at a constant rate of 9.8 m/s^2. In terms of our schema, then, 'ϕ' is 'system falling near the surface of the Earth,' and 'χ' is 'system accelerating at a constant rate of 9.8 m/s^2.' Now it is not true that all systems falling near the surface of the Earth accelerate constantly at 9.8 m/s^2. For one thing, there are feathers, leaves, and scraps of paper (like Neurath's thousand mark note[69]) whose fall is influenced by the passing breezes in such a way that their acceleration is often very different from the value cited in the law. Suppose we find some way to modify the law so as to exclude such bodies from consideration, and so as to restrict attention to objects such as bricks, hammers, cannonballs, and people, objects which, relatively speaking, are only marginally affected by non-gravitational forces.[70] I will suppose that this restriction can be effected in some principled manner, and indicate it by letting 'ϕ' stand for 'system falling *freely* near the surface of the Earth.'[71]

Even with this modification, however, it is not the case that all ϕ's are χ's. The problem is not that the number 9.8 is not exactly right, but that there *is* no right number. The acceleration of a freely falling body varies from place to place, and in two ways: at a given latitude, it decreases with the height of the body above the ground, and for a given height, it increases with latitude.[72] Thus the statement that all freely falling bodies near the surface of the Earth accelerate at a constant rate of 9.8 m/s^2 is straightforwardly false.[73]

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[69] See, e.g., Hempel (1969), p. 173.

[70] As will become clear, restricting the law in this way is a step towards the production of an idealized law (as opposed to a quasi-law). See especially the last paragraph of section 8.

[71] In the present context, then, the phrase 'free fall' is not intended to imply the complete absence of non-gravitational forces such as air resistance, but merely their relative negligibility. (Note, on the other hand, that even if the complete absence of any force other than the Earth's gravitational pull were required for free fall, everything I say in the next paragraph would still be true.) The hope is also, of course, that we have been able to arrive at a statement which is at least somewhat plausible without having to define the notion of free fall in such a way that we end up with a trivial truth.

[72] The first sort of variation is a straightforward consequence of Newton's Law of Universal Gravitational Attraction, which we can treat as a genuine law for the present purposes of illustration (!); the second is due to fact that the Earth is oblate rather than spherical. (For some data on this latter point, see Cohen(1985), p. 175.)

[73] Remember that for expository purposes I am assuming that such 'ϕ's are χ's' statements should be read as entailing the corresponding '$(\forall x)(\phi x \rightarrow \chi x)$' claims.

Despite its falsity, we often treat the statement in question as though it expresses a genuine law when providing explanations, making predictions, designing equipment, and so on. In doing so, we are idealizing, and the statement itself can properly be said to be an idealization. This, then, is an example of what I am calling a quasi-law. And it is interesting to note that each of the other three factors mentioned above seem to be present: the statement is a simplification of the truth of the matter, it is an approximation to that truth, and the features of the systems in question which are misrepresented will certainly be relevant features in the contexts in which we treat the statement as expressing a law.

As with models, we can coherently talk about the degree of idealization involved in a given quasi-law, and presumably that is a product of two factors: the degree of idealization (that is, degree of distortion) involved in saying, of each non-χ ϕ, that it is a χ, and the ratio of non-χ's to χ's amongst the ϕ's.[74]

7. Idealized Laws

With a quasi-law, misrepresentation is involved at each of three different levels: in treating particular ϕ's as χ's when applying the law to them; in making the general claim that ϕ's are χ's; and in the distinct claim that it is a law that ϕ's are χ's.[75] Given especially the presence of the second kind of misrepresentation, the general formula that idealization involves misrepresentation carries over quite straightforwardly from the case of models to the case of quasi-laws—a quasi-law says something false, and so misrepresents the world. When it comes to what I want to call "idealized laws," however, misrepresentation is involved only in a somewhat more indirect way. The idea

[74] Or, to use the mathematical metaphor introduced earlier, we can say that the degree of idealization enjoyed by the quasi-law can be calculated by taking a weighted sum over the non-χ ϕ's, with the weights representing the degree of idealization involved in representing the corresponding ϕ's as χ's, and then (to get a ratio) dividing by the total number of ϕ's (χ and non-χ).

[75] To see that these are distinct claims, we need only allow that it possible for the claim that ϕ's are χ's to true whilst the claim that it is a law that ϕ's are χ's is false.

here is that, while the statement which expresses an idealized law is perfectly true (and while it is true to say that it expresses a law), there is typically misrepresentation involved in the *employment* of the law, in that we often bring it to bear upon systems which are not ϕ's. When we know that we are idealizing, then, the difference between quasi-laws and idealized laws corresponds to the difference between two distinct sorts of pretence: we pretend that quasi-laws are laws, whereas we pretend that idealized laws apply more often than they do.[76]

To illustrate the notion of the idealized law, consider the law of inertia, which states that a body subject to no net force undergoes no acceleration, and which we will suppose to be a genuine law. The law of inertia is clearly a law which rarely, if ever, finds a foothold in the world, just because few, if any bodies have ever been subject to zero net force.[77] Nonetheless, we sometimes employ the law as though it applied to various actual bodies, regarding it as a good approximation for various purposes to treat those bodies as suffering no net force. And in so applying the law, we either implicitly or explicitly misrepresent the systems to which we apply it.

I will take an indirect approach to the task of making the notion of an idealized law more precise, by first asking what it might mean to say that there is idealization involved in a single instance of our employing a statement *L*, of form 'ϕ's are χ's,' with respect to some system *S* (thus focussing on what it means to idealize in using a law, rather than on what it means to say that a law itself is idealized). One possibility is that part of what is meant is that although *S* is a ϕ, it is not really a χ; another is simply that *L* is not true, or is not a law, regardless of the particular properties of *S*; but in any of these cases *L* at best expresses a quasi-law.[78] Another possibility, however, is that *L* expresses a

[76] To put it another way, with an idealized law we pretend that there are more ϕ's than there are.
[77] It might be objected that the law finds footholds aplenty if only we write it in the contrapositive form "All accelerating bodies are subject to a net force," for a multitude of accelerating bodies surrounds us. I will return to consider this potential objection at the end of this section.
[78] Given again my assumption, made for purely expository purposes, that 'ϕ's are χ's' should be taken to entail '$(\forall x)(\phi x \rightarrow \chi x)$.'

genuine law, but that *S* is not really a ϕ (and thus that we are implicitly or explicitly misrepresenting *S* by employing *L* with regard to it). Clearly that is not enough to make it appropriate to say that we are idealizing when we employ *L* in our treatment of *S*—sometimes we are simply flat wrong. Talk of idealization comes to seem more fitting if, in addition to its being the case that *S* is not a ϕ, it is also true that (i) *S* is approximately a ϕ, that (ii) it is a simplification of the truth about *S* to say that it is a ϕ, and/or that (iii) in describing *S* as a ϕ we are misrepresenting certain relevant features of the system (where relevance is then to be understood in one of the ways discussed earlier). More generally, if *L* is a genuine law, but it is an idealization to regard *S* as a ϕ—or, to put it another way, it is an idealization to regard *L* as applying to *S*—then we are idealizing by using *L* in our dealings with system *S*.

Given this, my suggestion is simply that we classify a law as idealized just in case we are typically forced to idealize in the way just described in employing the law, for the reason that true ϕ's are relatively rare. Summing up, then:[79]

> 'ϕ's are χ's' is, or expresses, an *idealized law* if and only if (i) it is a genuine law that ϕ's are χ's, but (ii) ϕ's are relatively rare, and so (iii) our employment of the law typically involves treating various non-ϕ systems as ϕ's even though it is an idealization to do so.

Thus if the ϕ's are plentiful, then we will not regard the law that ϕ's are χ's as an idealized law, but note that this it is still entirely possible that we will sometimes idealize in using the law, by employing it with regard to non-ϕ systems and idealizing them in doing so.

As with models and quasi-laws, it makes sense to speak of the degree of idealization exhibited by an idealized law—there can be highly idealized laws and laws which are only somewhat idealized. One obvious way in which this arises is via the phrase 'relatively rare' (and the connected quantifier 'typically'). The rarer the ϕ's, the

[79] The same remarks apply to this definition of the notion of an idealized law as I made with respect to the definition of the notion of a quasi-law—see n. 61.

more idealized the law. But note that the degree of rarity of the ϕ's cannot alone account for all the ways in which one law may be more idealized than another. Fortunately, however, idealizations (of particular systems, in specific respects) also come in degrees, and so how idealized an idealized law is will be a matter both of how rare the ϕ's are, and of how much of an idealization is involved when we treat various non-ϕ's as ϕ's in our employment of the law. Suppose, for example, that there is not now, never has been, and never will be an "untrammeled" body, that is, one subject to no net force. It follows that untrammeled bodies and perfectly spherical untrammeled bodies are equally rare; yet intuitively we might be tempted to classify a law governing perfectly spherical untrammeled bodies as more idealized than a law which concerns all untrammeled bodies regardless of shape. The relative degrees of approximation, simplification, and distortion of relevant features involved in the employment of the two laws might account for that temptation, and make it a respectable one.

One aspect of the definition of idealized law which perhaps calls for a little further comment is the notion of rarity invoked in the second necessary condition on being an idealized law, and how not to understand it. Specifically, rarity cannot here be tied to absolute number in the most straightforward way. Suppose that a certain kind of stellar event occurs once every billion years, on average, but that we are fortunate enough to avoid a Big Crunch, so that the universe lives on indefinitely into the future. In that case, there will eventually be a very large number of events of the kind in question; yet such events will surely still count as rare occasions. Instead, it seems to me that to understand rarity in this sense, we need to see the law statement in question as embedded in a wider context—as part of a theory, or as a representation employed in the context of a particular sort of theorizing. We can then draw on the notion of a domain of inquiry, a class containing just those systems which the theory is a theory of, or which the theorizing is about. The ϕ's then count as rare when a sufficiently small proportion of systems in the relevant domain of inquiry are ϕ's. To take a relatively extreme example, the bodies subject to a net force of zero are clearly a very small proportion of the class of all, and it is for that reason that the second necessary condition on being an idealized law is satisfied for the law of inertia. [80]

Given this way of understanding rarity, let me lay aside a potential objection which strikes me as misguided. The objection I have in mind is that the notions of "proportion" and of "sufficient smallness" on which the explication depend are unacceptably vague.[81] Such a complaint could be handled in a two-pronged way. If all that is sought is an account of the way in which we classify some laws as idealized, then some vagueness in the terms of the account is not obviously a bad thing—after all, there is plausibly some vagueness inherent in the classificatory practice itself. If, on the other hand, it should seem desirable to construct a notion of idealized law which is considerably more precise for some philosophical purpose or other, then the understanding the proposal at hand in this way certainly leaves room for such a development; perhaps the relevant notion of proportion might be cashed out in measure-theoretic terms, for example.[82]

It is also worth noting that the notion of an idealized law I have outlined might be, and indeed probably should be elaborated upon by taking into account the extent to which, in treating of non-ϕ systems by employing a 'ϕ's are χ's' law, thinking of the

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[80] Note that whether the ϕ's are rare will in many cases be a contingent matter—not merely logically or metaphysically contingent, but physically or nomologically contingent. This strikes me as an advantage of the account I am offering, for it seems to me that in at least some cases when we say that a law is idealized, we are (and take ourselves to be) uttering a (physically) contingent truth. (Had the world been full of ϕ's, as it might have been, such-and-such a law would not have counted as idealized; our use of the law constitutes an idealization only relative to the details of the particular world in which we employ it.) If there are cases, however, of idealized laws which do not seem to possess their status only contingently, then my hope would be that they qualify in virtue of there being sorts of rarity which are not physically contingent.

Incidentally, reflection on the notion of a quasi-law makes it immediately clear that whether *L* expresses a quasi-law is also contingent, but that, the presumed physical contingency of our theoretical practices being what they are aside, quasi-lawhood is only a metaphysically, and not a nomologically contingent matter—it is trivially true that whether *L* expresses a law or not depends on what the laws are.

[81] What is more, the domain of inquiry may well have vague boundaries.

[82] Comparing the sheer cardinality of the class of ϕ's with that of the class of χ's, however, will clearly not work, for as the example of rare stellar events in a universe of infinite lifetime suggests, both sets might have the cardinality aleph-nought even when the ϕ's do count as rare. The obvious problem a measure-theoretic approach would face, on the other hand, is that of locating a suitable provenance for the necessary measures.

relevant systems as χ's also constitutes an idealization, and thus by taking into account the extent to which 'χ' contributes to the classification of the law as idealized, and as less or more so. Given what has come before, elaboration of that sort would be a relatively straightforward matter, and I will not enter into such embroidery here.

Let us close this discussion of the notion of an idealized law by considering an objection which might be raised to the definition laid out above. The objection is this: According to the proposed definition, for 'ϕ's are χ's' to state an idealized law, the ϕ's—that is, systems of the type mentioned first in the candidate law statement—must be, at best, few and far between in the relevant domain of inquiry. Another way of stating the very same law, however, is simply to take the contrapositive of the initial formulation, that is, 'Non-χ's are non-ϕ's'; and it may be that, while the ϕ's are rare, the systems mentioned first in this new formulation of the law, the non-χ's, are quite plentiful, or even ubiquitous in the relevant domain. Thus it seems that whether a given law counts as an idealized law may depend on which of two syntactically distinct but semantically equivalent ways of expressing the law we consider. And surely that is wrong; surely we are here trying to classify *laws* (or putative laws, in the case of quasi-laws) with an eye to types of idealization, not law *statements*. Indeed, the law of inertia provides a perfect example: it may be that no body has ever truly been subject to no net force, and yet a multitude of accelerating bodies (non-non-accelerating bodies, so to speak) surrounds us. So is the law of inertia an idealized law or not?[83]

The first point to be made in response to this objection is that it relies on an assumption about law statements which by no means all accounts of lawhood would regard as legitimate. Specifically, the objection assumes that when 'ϕ's are χ's' is a law

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[83] Essentially the same line of thought just as readily gives rise to the claim that the definition I introduced earlier of what it is for a law or law statement *apply* to a system is ill-formed with respect to laws (as opposed to law statements): The law that ϕ's are χ's is supposed to apply to *S* iff *S* is a ϕ; yet (this line of thought goes), the law that non-χ's are non-ϕ's is the same law, and a given *S* may be a non-χ but not a ϕ, or vice versa. (Indeed, if the law in question really is a law, then on this view any given *S* is bound to be one of the two!) So does the law apply to such an *S* or not? This is the worry I alluded to in n. 54, and the discussion which follows in the text should make it clear how I would respond to it.

statement, a statement of the form 'Non-χ's are non-ϕ's' states the same law. This is straightforwardly true if the logical form of law statements can be captured by the first-order predicate calculus in the obvious way—'$(\forall x)(\phi x \rightarrow \chi x)$' and '$(\forall x)(\neg\chi x \rightarrow \neg\phi x)$' are indeed logically equivalent[84]—and so there is a difficulty to be faced in combining accounts of law which endorse that view (such as those due to Ayer (1956) and Lewis (1983, p. 366-7)) with the present proposals concerning the notion of an idealized law. On some other leading accounts of lawhood, however, the crucial assumption does not hold good. For example, if 'ϕ's are χ's' says that the universal ϕ-ness necessitates the universal χ-ness, then 'Non-χ's are non-ϕ's,' read as a law statement, would have to be understood as claiming that the universal non-χ-ness necessitates the universal non-ϕ-ness—a different claim, and one which at least some proponents of this view of laws would regard as not even being materially equivalent to the first (for the right choice of 'ϕ' and 'χ'), on the grounds that there is no such universal as "non-χ-ness" or "non-ϕ-ness."[85] Similarly, the claim that ϕ's, in virtue of being ϕ's, have the capacity to χ, is clearly distinct from the claim that non–χ's, in virtue of being non-χ's, have the capacity to non-ϕ.[86] Thus, the objection we are considering has no force unless we adopt the right (or wrong) account of lawhood.

What if the best account of lawhood is one on which 'ϕ's are χ's' and 'Non-χ's are non-ϕ's' state the same law, however? For strategic purposes, it would be preferable if the account I am presenting here of various types idealization in laws could continue to avoid commitment to any but the most minimal assumptions about the nature of laws.[87] With that goal in mind then, we could redefine the notion of an idealized law as follows:

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[84] This is also a very familiar point, of course, as it is the observation which gives rise to the ravens paradox in confirmation theory.

[85] Armstrong springs to mind here as someone who believes that there are universals, but relatively few of them. More to the point, Armstrong argues that when the predicate 'is ϕ' picks out a universal, the predicate 'is non-ϕ' typically will not. See Armstrong (1978), chapters 13 and 14.

[86] See nn. 51-53 above.

[87] Specifically, although Ayer's (1956) account seems deeply problematic to me (and many others), and although I am in fact inclined towards a "capacities account" of the sort Cartwright has been developing, I would rather not presuppose the falsity of what Earman calls the "Mill-Ramsey-Lewis" or "M-R-L" view

> *L* is an *idealized law* if and only if (i) *L* is a genuine law, (ii) there is a formulation of *L* of the form 'ϕ's are χ's' such that ϕ's are relatively rare, and (iii) we often employ the law in a way which involves treating various non-ϕ systems from the domain of inquiry as ϕ's even though it is an idealization to do so.

This new definition retains all the central features of the initial definition, but avoids the difficulty we have been considering involving contrapositive formulations of laws. In fact, the first and second clauses on their own are sufficient enough to defuse the objection involving contrapositives, strictly speaking—the law of inertia would now be declared an idealized law, given this formulation, even without (iii). But (iii) is included because without it we get the wrong results: if we only ever used the law of inertia to draw the conclusion about various accelerating systems that they must be subject to net forces, say, then there would be little reason to talk of idealization. In other words, it is because we *use* the law in a certain way that it counts as an idealized law. But note that this is quite parallel to the case of quasi-laws: to be a quasi-law, *L* has to be *treated as* a law for some purposes. And this pragmatic dimension to the conceptual distinctions I am drawing is present even in the case of idealization in models. The account of idealization in models took as one of its fundamental building blocks the notion of idealization in a specific respect, and that has partly to do with relevance, which as characterized is clearly a pragmatic notion, and partly to do with simplicity, which is quite plausibly pragmatic, too.

Despite all this, it might be argued that the new proposal still has its defects. As things stand, if the law statement 'All accelerating bodies are subject to a net force' does express the same law as 'A body subject to no net force undergoes no acceleration,' then it expresses an idealized law just because it expresses a law which, by this definition, counts as idealized (and which, in fact, turns out to be highly idealized). And that

(1986, pp. 87-90), and on that view the objection involving contrapositives would indeed have teeth, just because, as noted, laws on the M-R-L view do have the logical form traditionally ascribed to them.

former statement thus counts as expressing a highly idealized law even though there is an abundance of non-idealizing uses of it available to us—we do, after all, use it to conclude that various accelerating bodies must be subject to some net force. If that should seem too uncomfortable a way of talking, then at the end of the day it may be best to refrain from talking of laws themselves as idealized or otherwise, and to limit ourselves instead to thinking about idealizing uses of laws. Before leaping into such a rethinking of the territory, however, it is worth remembering that these latter difficulties arise only if certain views of the nature of lawhood should turn out to be correct.

8. Ideal laws

In an attempt to capture one more way in which we laws can be tied up with idealization, let me begin simply by defining the notion of an ideal law, as follows:

> *L*, of the form ϕ's are χ's, is an *ideal law* if and only if (i) *L* is a genuine law, and (ii) ϕ's are ideal systems.

The first question which comes to mind here is what it means to say that the ϕ's are "ideal systems." The basic idea is that some systems (or possible kinds of system) are ideal in the sense of being perfect, or "just right," and that some laws then count as "ideal" just because they govern such perfect systems (or would govern such systems if there were any). But what makes perfect systems perfect?

Cartwright proposes an intriguing answer to that question. In essence, she proposes that certain sorts of system count as perfect or ideal because conditions are just right for some particular capacity to reveal itself in such systems, without hindrance from any distinct factor which might otherwise interfere.[88] Thus a body subject to no net

[88] Cartwright (1989), chapter 5, esp. pp. 190-1. Ideal laws as I am characterising them would seem to correspond to the laws which, on Cartwright's account (and in her terms), describe what happens in a situation depicted by an idealized model. Cartwright says that a law of this sort is "a kind of *ceteris paribus* law: it tells what [a] factor does *if* circumstances are arranged in a particularly ideal way" (*ibid*., p. 192). I do not wish to presuppose, however (and nor, I suspect, does Cartwright), that no ideal law ever applies to an actual system; perhaps ideal conditions are sometimes realized.

force is one which will reveal the inherent, if relatively unexciting capacity every body has to just keep moving with a constant velocity.[89] I find Cartwright's proposals attractive, and will implicitly allow them to fix the sense of the phrase 'ideal system' in the remainder of the discussion, but it is worth bearing in mind the fact that one might wish to consider some other sense in which a system could be perfect, or ideal; the hope is then that the rest of what I have to say will carry over to other ways of being ideal.

The next issue to be addressed here is the relationship between the category of ideal laws and that of idealized laws, and we can become clearer on the nature of that relationship by thinking first about the connections between being rare and being ideal. Logically speaking, these would seem to be independent features of a system (or possible kind of system). Certainly some kinds of system are both rare and ideal: consider the category of untrammeled bodies. It is just as surely true, however, that there are kinds of system which are rare without being ideal (in any obvious sense)—charged metallic spheres subject to a gravitational force of magnitude 10.378 N and a repulsive electrical force of magnitude 17.58 N pushing in directions which make an angle of 53° with each other, for example—and it also seems possible that the world might have been overflowing with ideal systems. Thus it is clear that the two features, being rare and being ideal, are quite separate.

On the other hand, it does seem to be true that in the actual world, ideal systems are rare—the two features are contingently correlated.[90] Suppose for a moment that it is true to say that *all* ideal systems are rare. Suppose, furthermore, that all ideal laws, of the form 'ϕ's are χ's,' are typically employed in a way which involves treating various non-ϕ systems from the domain of inquiry as ϕ's, even though we are idealizing those systems in doing so. Then it would follow that all ideal laws are idealized laws.

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[89] Cartwright may prefer to classify this as a (mere) tendency rather than a capacity (*op. cit.*, p. 226), but that distinction need not concern us here.
[90] And note that Cartwright's proposals account for this fact quite straightforwardly, given the evident fact that our world is a rich and complex one in which numerous causal factors are typically at work in any given situation.

The truth, of course, may not be so simple. Perhaps there are some kinds of ideal system which are relatively common. Or (more likely) perhaps there are ideal laws which we rarely or never use in the idealizing way described—simply because we never use the laws in question at all, or because the idealization involved in treating the actual non-ϕ systems around us as ϕ's is simply too great for it ever to be useful for us ever to employ the law in our dealings with non-ϕ systems. Even so, it seems clear that there is considerable overlap between the categories of ideal law and idealized law. We do in fact often use ideal laws which apply (or would apply) only to some rare sort of ideal system as though they applied to various non-ϕ systems in the world, and idealize in so employing such laws. The law of inertia again provides us with a good example.

With this in mind, we can now see that ideal laws are implicated in practices of idealization a little more loosely than either quasi-laws or idealized laws. Even if it is true that all ideal laws are in fact idealized laws, this is so only contingently. It is no part of the definition that an ideal law need ever be used in a way which involves idealizations, and this distinguishes ideal laws from both quasi-laws and idealized laws. Nonetheless, contingent though it may be, the overlap between the categories of ideal law and idealized law is there, and so *de facto* a full account of how idealizations arise in our use of laws must take account of the category of ideal laws.[91]

There is one problem to be dealt with here before leaving ideal laws behind—the worry about contrapositives again. Is law of inertia an ideal law or not? Untrammeled bodies are plausibly to be thought of as ideal systems (at least insofar as they are untrammeled), but an accelerating body does not thereby strike one as particularly ideal

[91] One might wonder why I have defined the notion of ideal law in such a way that the connection between ideal laws and idealizations is so loose. The answer is that it seems to me that we do pre-philosophically recognise a category of laws which are special just because they deal with systems which are ideal in some sense; and although it is true that such laws tend to get used in ways which involves us in idealization, that is not necessarily part of what we have in mind when we label such laws "ideal laws" or "laws concerning ideal systems." In other words, doing things this way seems to me to result in a greater degree of continuity with pre-existing usage.

Incidentally, another standard locution which comes to mind when we think about idealization in laws is that of the ϕ's being a "special case"; this, it seems to me, is ambiguous between the notions of being rare and being an ideal system, and so between talk of an idealized law and an ideal one.

in any obvious sense. So which formulation of the law of inertia should we look to when asking whether the law is an ideal one?

Given the discussion of the parallel worry at the end of the last section, it is easy enough to see what we might say here. For one thing, we might challenge the claim that the contrapositives of law statements are semantically equivalent to the law statements we started with.[92] More diplomatically, we might modify the definition of the notion of an ideal law, without losing anything essential, as follows:

> *L* is an *ideal law* if and only if (i) *L* is a genuine law, (ii) there is a formulation of *L* of the form 'ϕ's are χ's' such that ϕ's are ideal systems, and (iii) we often employ the law in a way which involves treating various systems as ϕ's.

Although the strategy here is in many ways parallel to the strategy I outlined for dealing with the contrapositives problem in the case of the notion of an idealized law, it is important to note that the third clause of this definition is not quite the same as the third clause of the amended definition that notion; in particular, it is not required that to be an ideal law *L* must be employed in such a way as to idealize non-ϕ systems, for it may be that whenever we employ the law in a way which involves treating various systems as ϕ's, the systems in question *are* ϕ's. As the discussion above makes clear, this is simply in keeping with the original definition of the notion of an ideal law. However, this new definition does require that we sometimes use the law, unlike the initial definition, for it is that usage which now puts an emphasis on the fact that the law can be thought of as concerning ideal systems.

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[92] Note that one can deny this without denying that it *follows from* the law of inertia (here identified as the law that a body subject to net force does not accelerate) that all accelerating bodies are subject to a net force. If, for example, the law of inertia is understood as concerning a relation of necessitation which holds between two universals, then given the law the contrapositive will surely be true; it is just that the contrapositive will not state the law of inertia (or, most likely, any law).

This completes my account of the specific ways in which laws and our employment of them can involve idealization. Before I turn to consider laws and abstraction very briefly, however, there are two further points worth making regarding idealization and laws.

First, there is the option of introducing further categories into our scheme for classifying laws and law statements. Ideal and idealized laws must genuinely be laws, whereas quasi-laws must not. Yet perhaps there are statements of the 'ϕ's are χ's' form which purport to govern rare or ideal systems, and which we treat as expressions of law in at least some contexts, but which do not in fact express genuine laws.[93] Some such statements might be said to express, or to be, ideal or idealized quasi-laws ('ideal' for ideal ϕ's, 'idealized' for ϕ's which are rare but non-ideal).[94] Employment of either an idealized quasi-law or (assuming that the ideal systems in question are few and far between) an ideal quasi-law will then typically involve us in the sorry business of making *both* the idealization that the statement in question is true *and* the idealization that it applies to the system in hand.[95]

Second, the classificatory scheme I have laid out corresponds nicely to an important claim Cartwright makes in her 1983 book, *How the Laws of Physics Lie*.[96] According to Cartwright, the sorts of law statement which we value most in our scientific work must generally be taken in one of two ways: as widely applicable but false, or as true but quite restricted in their scope of application. The idea is that a typical law statement of the form 'ϕ's are χ's' will have numerous exceptions if it is read as entailing '$(\forall x)(\phi x \rightarrow \chi x)$,' and so as applying to all ϕ's, and that in the subsequent

[93] And for reasons having to do with the falsehood of what I earlier called their "extensional content"; see n. 67.
[94] "Some" because, in line with the preceding discussion, the conditions just specified may not be enough to make something count as an *idealized* (as opposed to ideal) quasi-law—for that, it needs to be the case that the quasi-law in question is typically employed in a way which idealizes non-ϕ systems by treating them as ϕ's.
[95] An example of an idealized quasi-law, in fact, is Galileo's law of free fall, as ordinarily understood and employed (not, that is, understood in terms of the special sense of 'free' I introduced towards the end of section 6).
[96] See especially essay 2, "The Truth Doesn't Explain Much" (Cartwright, 1983).

attempt to produce a true statement of the '($\forall x$)(....)' form we will find ourselves having to add a slew of restrictive antecedent clauses, and possibly even resorting to the use of non-specific *ceteris paribus* clauses (thus effectively replacing 'ϕ' with some new 'ϕ^*'); the end result will at best be a true generalization of very restricted applicability, one which will do very little of the work we expected our original law statement to do. Cartwright illustrates this dilemma by employing the example of Snell's law of refraction.[97] In the terms of my account, Cartwright's claim is that our favorite law statements must generally be taken to express either quasi-laws, or idealized (and possibly ideal) laws, so that we must either sacrifice truth or applicability. Cartwright's main point is that if this is claim is correct, it is quite devastating for covering-law models of explanation, but it is also worth reflecting on the fact that the claim raises serious problems for a wide range of accounts of confirmation in much the same way.

9. Abstraction in laws

I wish to say relatively little about abstraction and laws. It is surely true of any law statement of the form 'ϕ's are χ's' that when we employ it in the treatment of a system *S*, describing *S* as being both a ϕ and a χ will omit mention of many features *S* has, without thereby misrepresenting *S* (that is, without representing *S* as lacking them). To classify *L* as an abstract law is thus presumably to imply that it involves, in one way or another, a lot of omission as compared to other laws. Accordingly, I propose that we understand talk of abstract laws in a way which derives from the notion of an abstract model (where an abstract model is one which omits a lot). Corresponding to any law statement of the form 'ϕ's are χ's,' there are two models of any given real system *S*, one of which simply represents *S* as being ϕ, and the other of which simply represents it as being χ. A law statement (or law, if the statement indeed expresses one) then counts as abstract if and

[97] *Ibid*.

only if, given an arbitrary *S* from the relevant domain of inquiry, one or both of the models in question is an abstract model of *S*. (Another way of putting this, perhaps, is that *L* counts as an abstract law just when one or both of the concepts ϕ-ness and χ-ness is a relatively abstract concept.) And talk of degrees of abstraction with respect to laws can then be understood in a way which derives fairly straightforwardly from our understanding of such talk with respect to models.[98]

Two aspects of this simple proposal concerning abstraction should be noted. First, whether a given law counts as abstract or not is determined with reference to an arbitrary system from the relevant domain of inquiry. This is simply because if a model the content of which is captured by '*S* is ϕ' (say) counts as an abstract one, where *S* is any system from the relevant domain of inquiry, then so too will the model the content of which is captured by '*S** is ϕ,' for any other system from that domain, *S**.

Second, note that the proposal in no way precludes the classification of a law (or law statement) as abstract and ideal, abstract and idealized, or as abstract and a quasi-law. In particular, although it is indeed built into my own account of abstraction in models that abstraction with regard to a particular feature of real systems involves omission *without* misrepresentation, the misrepresentation which is prohibited is just misrepresentation of the fact that the real system or systems in question have the feature in question; nothing in the account prevents an abstract model from simultaneously misrepresenting how things stand with respect to *other* features of systems (features other than the ones the model abstracts away from).[99]

The brevity and apparent simplicity of this discussion of abstraction and laws should not deceive: all the work is done by the notion of an abstract model, and as the

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[98] The only obvious complication being that the degree of an abstractness of the law will be a product (loosely speaking) of the degree of abstractness of the two models to which it gives rise (the ϕ model and the χ model).

[99] It is, perhaps, especially important to note that the account allows for abstract quasi-laws in the context of Cartwright's views, for we might expect a law statement which is abstract in part because the concept of ϕ-ness is an abstract concept to have a wide range of application, and thus, according to Cartwright, expect it to be false.

earlier discussion of that notion made plain, providing an account of the classification of models as more or less abstract is no trivial matter. There are also a good number of difficult questions about how abstract laws function, how precisely they should be understood, and what epistemological status they have.[100] The hope is, nonetheless, that the framework laid out in this paper will help us as we grapple with such questions about idealization, abstraction, and the implications of their ubiquity for our philosophical understanding of the sciences.[101]

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[100] See also Cartwright (1989), chapter 5, for much more on these issues.

[101] Thanks for helpful conversations to Mauricio Suárez, Mathias Frisch, Nancy Cartwright, Paul Teller, R.I.G. Hughes, Paddy Blanchette, Paolo Mancosu, Dorit Ganson, and Peter McInerney.

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